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Group-velocity dispersion and deterministic PMD of modes in a hollow omnidirectional Bragg fiber.

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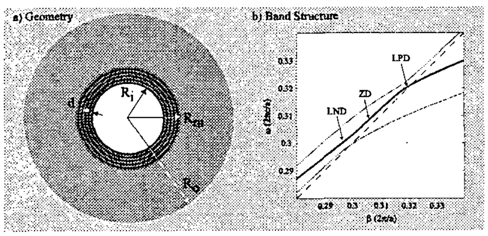

Fig. 1. (a) Schematic of omnidirectional photonic Bragg fiber. Hollow core of radius R_i, mirror region of N bilayers of thickness d, surrounded by an over-cladding extending to R_o. (b) Typical band structure of an OPBF. The diagonal dashed line is the air light line. To the left of the light line are guided modes of the core, while to the right are surface states of the mirror. The thick curve corresponds to the HE_{11} mode, exhibiting regions of large negative dispersion (LND), zero dispersion (ZD), and large positive dispersion (LPD). Thin lines are other nearby modes.

Omnidirectional photonic bandgap fiber (OPBF) is a new class of Bragg fiber based on omnidirectional reflectivity [1]. The large hollow core of these fibers and the large index contrast in the surrounding omnidirectional mirror make the fiber support a number of low-loss core-guided modes. Through deliberate fiber design one can tailor the fiber modes to exchange properties with other fiber modes over narrow frequency ranges (avoiding crossing), which gives the fibers rich, controllable dispersion properties. The dispersion properties of the OPBF fiber can be tailored very accurately, because of the large number of degrees of freedom the fiber geometry possesses. This makes OPBF fibers good candidates for long-haul transmission, dispersion compensation, zero-dispersion transmission, or other applications that require accurate dispersion control. Intimately connected to the mode dispersion properties is the detrimental effects of Polarization Mode Dispersion (PMD). When dealing with potential applications like long-haul transmission or dispersion compensation, one must ensure that the PMD of a fiber is acceptably small. The aim of this work is to demonstrate dispersion and PMD properties of the modes of OPBFs.

OPBF consists of a hollow core, a series of bilayers of contrasting refractive-index glasses, and an over-cladding. Figure 1a illustrates the cross section of an OPBF. Modal electromagnetic fields decay exponentially in the mirror. Thus, field penetration into the mirror is largely limited to the first few bilayers. Moreover, the modal dispersion relation is very sensitive to the first several mirror layers. In fact, this sensitivity allows group-velocity dispersion control by design of such layers.

For a particular realization of OPBF, the band structure of a doubly degenerate HE_{11} mode is shown in Figure 1b. Data is presented in dimensionless units where a sets the length scale, ω is frequency, and β is propagation constant. Because of the avoided crossing with other guided modes, the HE_{11} dispersion curve exhibits regions of large negative dispersion, zero dispersion, and large positive dispersion.

Quantitative analysis of a mode's dispersion properties involves calculating a modal dispersion relation $\omega(\beta)$. For cylindrically symmetric dielectric fiber profiles, this can be accomplished by a well established transfer matrix technique [1]. Evaluation of the deterministic PMD (arising from uniform along the direction of propagation perturbations) is a much harder problem as it involves the change in the dispersion relation of an originally doubly degenerate mode when the waveguide geometry is perturbed away from cylindrical symmetry. In the case of low index-contrast waveguides, the problem of deterministic PMD evaluation was successfully solved in the context of coupled-mode theory in [2]. However, this formulation was found to fail in the case of high index-contrast. Perturbation and coupled-mode theory formulations for evaluation of deterministic PMD for a generic class of geometric perturbations of high index-contrast dielectric waveguide profile have been developed by the authors in [3–5]

In the following, we apply this formulation to characterizing PMD of a doubly degenerate mode of an OBPF for a common elliptical perturbation of a fiber profile. We establish that, if in some range of frequencies a doubly degenerate mode of angular index $m = 1$ behaves like a mode of pure polarization TE or TM (where polarization is judged by the relative amounts of the electric and magnetic longitudinal energies in a modal crossection), its inter-mode dispersion parameter (which defines a deterministic PMD) $\tau = \frac{\partial \triangle \beta_e}{\partial \omega}$ is strongly correlated to the group-velocity dispersion D by $\tau = -\lambda \delta D$, where δ is a measure of the fiber ellipticity and $\triangle \beta_e$ is a split in the propagation constant of a linearly polarized doubly degenerate mode due to an elliptical perturbation. This indicates that regions of high dispersion will, generally, correlate with regions of high PMD. Thus, fiber design needs to be optimized to reduce PMD value when high group-velocity dispersion is desired.

We calculate the PMD of elliptically distorted OBPFs by using a Hamiltonian formulation of Maxwell's equations in terms of transverse components of electromagnetic

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fields [1,3,4]. In the case of a uniform circularly symmetric waveguide profile, modes (field solutions of Maxwell's equations) will satisfy a generalized Hermitian eigenvalue problem

$$\beta\hat{B}\,|\psi^0_\beta\rangle = \hat{A}_0\,|\psi^0_\beta\rangle\,, \tag{1}$$

with the following orthogonality relation between the modes

$$\langle\psi^0_{\beta^*}|\hat{B}|\psi^0_{\beta'}\rangle = \frac{\beta'}{|\beta'|}\delta_{\beta,\beta'}. \tag{2}$$

When a uniform perturbation of a waveguide profile is introduced into a system, the operator $\hat{A}_0$ will be modified. We denote the correction to an original operator $\hat{A}_0$ as $\Delta\hat{A}$. Then, eigenproblem (1) is modified, becoming

$$\tilde{\beta}\hat{B}\,|\psi\rangle_{\tilde{\beta}} = (\hat{A}_0 + \Delta\hat{A})\,|\psi\rangle_{\tilde{\beta}}\,. \tag{3}$$

We consider a general scaling perturbation that is uniform along $\hat{\mathbf{z}}$ axis, where the discontinuous dielectric interfaces of unperturbed radii ρ_i are described, when perturbed, by a new set of curves $x_{scaled} = \rho_i Cos(\theta)(1+\delta_x)$, $y_{scaled} = \rho_i Sin(\theta)(1+\delta_y)$, where $\theta \in (0,2\pi)$, $i \in (1, Number\ of\ interfaces)$. The case of $\delta_x = \delta_y = \delta$ corresponds to a uniform scaling, while the case of $\delta_x = -\delta_y = \delta$ corresponds to a uniform ellipticity of a waveguide profile. New eigen values $\beta^\pm$ of the split doubly degenerate eigen mode are found by solving standard secular equations

$$\beta^\pm = \beta + \frac{<\psi_{\beta,m}|\triangle\hat{A}|\psi_{\beta,m}>}{<\psi_{\beta,m}|\hat{B}|\psi_{\beta,m}>} \pm \frac{<\psi_{\beta,m}|\triangle\hat{A}|\psi_{\beta,-m}>}{<\psi_{\beta,m}|\hat{B}|\psi_{\beta,m}>}. \tag{4}$$

PMD is defined to be proportional to the inter-mode dispersion parameter, which in terms of the group velocities mismatch is $\tau = \frac{1}{v_g^+} - \frac{1}{v_g^-}$. This can also be expressed as $\tau = \frac{\partial(\beta^+-\beta^-)}{\partial\omega} = \frac{\partial(\triangle\beta_e)}{\partial\omega}$.

In [4] we derive the following expressions for $\triangle\hat{A}$. The case of a uniform scaling perturbation $\delta_x = \delta_y = \delta$ gives

$$\triangle\beta_s = <\psi_{\beta,m}|\triangle\hat{A}|\psi_{\beta,m}> = 2\delta\omega\int_S ds(\epsilon|E_z|^2 + |H_z|^2)\,, \tag{5}$$

where integration is performed over the fiber cross section. Moreover, as shown in [4], $\triangle\beta_s$ defines group-velocity dispersion through the following equalities $\triangle\beta_s = \delta(\omega\frac{\partial\beta}{\partial\omega} - \beta)$, $\frac{\partial\triangle\beta_s}{\partial\omega} = \delta\omega\frac{\partial^2\beta}{\partial\omega^2} = -\lambda\delta D$.

Next, consider the case of a uniform-ellipticity perturbation $\delta_x = -\delta_y = \delta$. The first-order correction to the split in the values of propagation constants of the modes $(\beta, m=1)$ and $(\beta, m=-1)$ due to the uniform re-scaling (4) is

$$\begin{aligned}&\triangle\beta_e = 2<\psi_{\beta,1}|\triangle\hat{A}|\psi_{\beta,-1}> = \\ &2\delta\omega\int_S ds[(-\epsilon|E_z|^2 + |H_z|^2) + 2Im(\epsilon E^*_r E_\theta - H^*_r H_\theta)],\end{aligned} \tag{6}$$

where E's and H's are the electromagnetic fields of the $(\beta, m=1)$ mode. In general, we find that for high index-contrast waveguides $\triangle\beta_e$ is dominated by the diagonal term $\sim \int_S ds[(-\epsilon|E_z|^2 + |H_z|^2)$, while for low index contrast waveguides the cross terms of (6) become equally important.

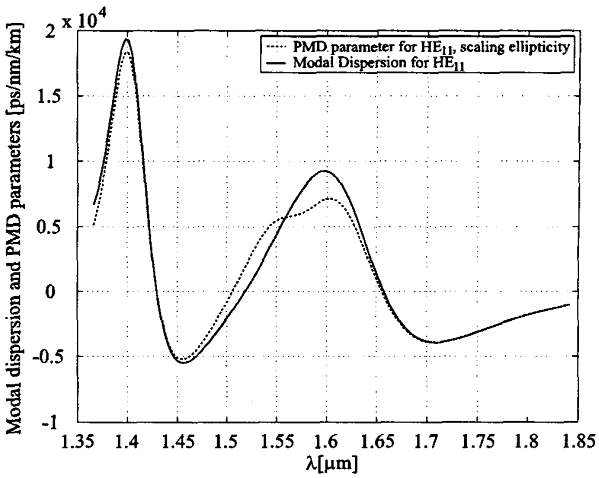

Fig. 2. HE_{11} group-velocity dispersion D and PMD parameter for a uniform ellipticity perturbation. PMD tends to positively correlate with dispersion, especially in the high dispersion regions. However, low value of PMD, and considerable value of group-velocity dispersion can be simultaneously achieved by fiber design. At $\lambda = 1.51\mu m$, for example, PMD is zero while $D = -2000\frac{ps}{nm\ km}$.

An important conclusion about PMD of a fiber can be drawn when electric or magnetic longitudinal energy dominates substantially over the other [4]. In the case of pure-like TE ($\int_S ds\epsilon|E_z|^2 \ll \int_S ds|H_z|^2$) or TM ($\int_S ds|H_z|^2 \ll \int_S ds\epsilon|E_z|^2$), the mode split due to the uniform scaling (5) becomes almost identical to the split in the degeneracy of the modes due to the uniform ellipticity perturbation (6). Thus $\triangle\beta_s \simeq \triangle\beta_e$. As PMD is proportional to $\tau = \frac{\partial\triangle\beta_e}{\partial\omega}$, and taking into account expressions for the frequency derivatives of $\triangle\beta_s$ we arrive at the conclusion that for such modes PMD is proportional to the group-velocity dispersion of a mode

$$\tau = \frac{\partial\triangle\beta_e}{\partial\omega} \simeq \frac{\partial\triangle\beta_s}{\partial\omega} = -\lambda\delta D. \tag{7}$$

In Figure 2, we present the HE_{11} group-velocity dispersion D and a PMD parameter defined as $-\frac{\tau}{\lambda\delta}$ for a uniform ellipticity perturbation and a particular design of an OBPF. As predicted by (7), the PMD of the mode follows the group-velocity dispersion closely, especially in the high-dispersion regions around $1.4\mu m$, $1.45\mu m$, $1.6\mu m$ and $1.7\mu m$. In the moderate-dispersion region around $1.51\mu m$, PMD and dispersion can be decoupled so that a low value of PMD and a still considerable value of the modal geometric dispersion $-2000\frac{ps}{nm\ km}$ are achieved.

References

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