Planar Porous Components for Low-Loss Terahertz Optics

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There is a strong interest in using the terahertz (THz) frequency band for applications in sensing, imaging, and wireless communications. To enable many of these applications, compact low-loss components for beamforming are required. Typically, such components are made using solid dielectric elements with spatially variable thickness, for example, planovconvex lenses or spiral phase plates. However, as losses in dielectrics typically greatly increase with THz frequency, so do the losses of the solid components. This work demonstrates that when introducing low-refractive index, low-loss subwavelength inclusions (air holes) into a solid material matrix, the loss of porous components can be greatly reduced compared to the loss of solid components with otherwise identical optical properties, thus opening a way to create efficient optical components even with nominally high-loss materials. Additionally, porous optical components can be created completely flat as spatially dependent optical path difference is achieved by varying the local porosity rather than the component thickness. This offers additional advantages for free-space alignment and integration of such components into optical systems. As an example, the design, fabrication, and experimental characterization of planar lenses and planar orbital angular momentum phase plates are carried out. It is then demonstrated how these porous components outperform their all-solid counterparts.

1. Introduction

Recently, there has been a strong interest in using the terahertz (THz) frequency band (from 100 GHz to 10 THz, wavelengths of 3 mm to 30 μm) for various applications in sensing,[1] imaging,[2] and wireless communications.[3] To enable many of these applications, low-loss beamforming components, such as lenses, polarizers, phase plates, beam steersers, etc., are needed.[4] Such devices generally operate by spatially modifying the phase of the incident plane wave. For example, for thin optical components located in the plane \((x,y)\) made of an optically uniform material of refractive index \(n\) and with a spatially dependent thickness \(h(x,y)\), the local phase incurred by the wave due to the passage through the component is given by

\[
\phi(x,y) = \frac{2\pi}{\lambda} n \cdot h(x,y)
\]

where \(\lambda\) is the wavelength of light. Thus, by judiciously designing the component thickness \(h(x,y)\), the wavefront of the plane wave can be modified at will. For example, in simple convex dielectric lenses, the lens thickness is varied quadratically with the distance from the lens center which allows focusing of parallel beams. In the THz range, commercial lenses are available with materials such as TPX (polymethylpentene), PTFE (Teflon), and high-density polyethylene (HDPE). However, for dielectrics in general, the absorption losses increase quadratically with the frequency. For example, one of the lowest loss commercial materials—TP features a power absorption coefficient of \(0.2 \text{ cm}^{-1}\) at 0.5 THz, while it increases to \(0.7 \text{ cm}^{-1}\) at 2 THz. Alternatively, PTFE and HDPE are other popular materials with power absorption coefficient of \(1.5 \text{ cm}^{-1}\) and \(2.1 \text{ cm}^{-1}\) at 0.5 THz, respectively, and 2.5 cm\(^{-1}\) at 2 THz.[5,6] Therefore, the conventional approach of designing beamforming components with variable thickness profile becomes challenging when increasing the THz operation frequency.

To overcome the loss challenge, one can use thinner optical components. Particularly, since the phase in Equation (1) can be reduced (wrapped) into a \((0,2\pi]\) interval, one can design thinner components with saw-like profiles that will have very similar performance to the components with a full \((0,2\pi]\) phase profile given by Equation (1).[7,8] However, these types of lenses—also known as Fresnel lenses—tend to show narrowband operation because of the chromatic dispersion of the underlying dielectric materials, and as a consequence, necessity of using different saw-like dielectric profiles at different frequencies.

Another method to design beamforming components is based on gradient index (GRIN) optics, which utilizes a...
nonuniform spatial distribution of the RI $n(x, y)$ with a fixed thickness $h$ to modify the local phase incurred by the wave

$$\phi(x, y) = \frac{2\pi}{\lambda} n(x, y) h$$  \hspace{1cm} (2)

Such components can be realized using metamaterials that consist of metals or dielectrics resonators engineered at a wavelength or subwavelength scale and patterned to produce an RI distribution at a larger scale.\(^\text{[28,29]}\) Thanks to the micrometer scale of the THz wavelength, THz metamaterials are easier to manufacture in the THz band than in the visible range. Recently, several designs of metamaterial beamforming devices have been proposed for the THz range, including lenses,\(^\text{[10–15]}\) defectors,\(^\text{[16–18]}\) polarization controllers,\(^\text{[19]}\) orbital angular momentum phase plates,\(^\text{[20]}\) beam splitters,\(^\text{[21]}\) etc. Typically, these metamaterial devices function via a resonant mechanism that can lead to high ohmic losses in metallic metamaterials, as well as relatively narrowband operation.

Alternatively, one can use structured materials with spatially nonuniform distribution of deep subwavelength inclusions to design effective materials with spatially variable RI. In this case, effective medium theories can be used to predict structured material RI and losses. For example, this methodology was successfully employed in the THz frequency band to design low-loss porous waveguides\(^\text{[22–27]}\) with elongated subwavelength air inclusions in a waveguide dielectric matrix. This type of structure was also used by Park et al. and Brincker et al.\(^\text{[28,29]}\) to design compact porous lens to replace the traditional hyperhemispherical lens of THz photoconductive antennas. In these works, silicon was chosen as the bulk medium due to its very low losses and dispersion in the THz range as well as its compatibility with GaAs growth process.

In this work, by extending this idea further, we show that using porous materials (with subwavelength porosity), it is possible to design low-loss THz bulk components even with nominally high-loss bulk materials. More importantly, we demonstrate that, for the same base material, porous optical components outperform their all-solid counterparts in terms of losses, while having otherwise identical optical characteristics. This is a nontrivial finding as porous materials tend to have lower refractive indices than their solid counterparts, therefore, to achieve identical optical characteristics, porous optical components tend to be thicker than the solid ones. Nevertheless, we find that with increased porosity, reduction in the porous material refractive index is less pronounced than reduction in the effective material loss, which is due to field enhancement inside of the low-RI, low-loss subwavelength inclusions. We believe that this work paves a way for building novel low-loss bulk and freeform optical components from otherwise high-loss functional materials (ultrahigh dielectric constant materials, highly dispersive materials, etc.), which is of great importance for many practical applications in THz imaging and communications.

As follows from the effective medium model within the Bruggeman approximation,\(^\text{[34]}\) when introducing zero-loss low-RI subwavelength air inclusions, the structured material losses reduce much faster with the porosity than the real part of the effective material RI. Thus, using the same dielectric base material, one can design porous materials that result in identical phase change (see Equation (2)) as that given by the solid materials (see Equation (1)), while having considerably lower absorption losses than their solid counterparts.

In this work, we first use the effective medium theory and finite element calculations to design several planar porous optical components that include a lens and an orbital angular momentum (OAM) phase plate and show that their losses are lower than those of their all-solid counterparts. We then fabricate these components using laser cutting technique (subtractive manufacturing). Finally, we characterize the fabricated component optical properties and compare them to the theoretical predictions. This paper is structured as follows. First, in Section 2, we present the numerical design of the low-loss THz components. Then, in Sections 3 and 4, we fabricate and characterize the planar lens and the OAM phase plate, respectively. In particular, OAM is a well-established property of light that manifests itself as a helical wavefront with an azimuthal phase term. OAM states have been recently explored as additional spatial division multiplexing (SDM) technique for wireless communications\(^\text{[30–32]}\) and are therefore of particular interest in the THz range.

2. Design of Low-Loss Porous THz Components

The planar porous components considered in this work are designed by varying the local effective RI via introduction of low-RI subwavelength inclusions ($n_m$, for example air) into the higher RI host material ($n_a$). When the inclusion size is smaller than the wavelength of light, the THz wave will experience an effective material RI $n_{\text{eff}}$ with a value bounded between $n_a$ and $n_m$. Experimentally, air inclusions can be formed either by using additive manufacturing techniques such as 3D printing, or subtractive techniques such as CNC drilling or laser cutting. Within this work, we explore laser cutting technique for the fabrication of planar porous components with inclusions in the form of cylinders of subwavelength diameter (see Figure 1a).

To model the complex effective RI of such materials, we first use Bruggeman model, which allows us to derive several simple analytical expressions for material RI and losses in the limit of high porosity. We then confirm the validity of the Bruggeman model by comparing its predictions to the effective RI and losses extracted from guided wave simulations in the infinitely periodic media with cylindrical inclusions performed using finite element COMSOL software.

In the following derivations, we assume that subwavelength inclusions are loss-less and made of air, with the correspondent dielectric permittivity

$$\varepsilon_a = n_a^2 = 1$$ \hspace{1cm} (3)

At the same time, we assume that the host material is lossy and characterized by the power absorption coefficient $\alpha_{\omega_m}$ meaning that the power decay with propagation distance is $\sim \exp(-\alpha_{\omega_m}z)$. This also means that the host material can be described by the following complex dielectric permittivity

$$\varepsilon_a = (n_m + ik_{\omega_m})^2 = \left(n_m + i\alpha_{\omega_m} \frac{\lambda}{4\pi}\right)^2$$ \hspace{1cm} (4)
where $n_m$ and $k_m$ are the real and imaginary parts of the material RI. Using the Bruggeman model for cylindrical inclusions\textsuperscript{[24]}

\[ f \frac{\varepsilon_a - \varepsilon_{\text{eff}}}{\varepsilon_a + \varepsilon_{\text{eff}}} + (1 - f) \frac{\varepsilon_n - \varepsilon_{\text{eff}}}{\varepsilon_n + \varepsilon_{\text{eff}}} = 0 \]  

(5)

we can express the transverse component (with respect to the axis of the cylindrical inclusions) of the effective permittivity as

\[ \varepsilon_{\text{eff}} = \sqrt{\left(\frac{1}{2} - f\right)(\varepsilon_a - \varepsilon_m)^2 + \varepsilon_a \varepsilon_m - \left(\frac{1}{2} - f\right)(\varepsilon_a - \varepsilon_m)} \]  

(6)

where $f$ is the filling fraction of inclusions by area. From this expression, we can now define the real $n_{\text{eff}}$ and imaginary $k_{\text{eff}}$ parts of the porous component effective RI, as well as its effective absorption coefficient by power $\alpha_{\text{eff}}$ by noting that

\[ \varepsilon_{\text{eff}} = (n_{\text{eff}} + ik_{\text{eff}})^2 = \left(n_{\text{eff}} + i\alpha_{\text{eff}} \frac{\lambda}{4\pi}\right)^2 \]  

(7)

It is important to reiterate that the Bruggeman model (Equation (5)) gives an effective value of the transverse component of the material dielectric permittivity tensor. Therefore, when using only this value for the material effective permittivity, one assumes that the electromagnetic field vector in the medium is directed mostly perpendicular to the axis of the cylindrical inclusions. This is indeed correct within the paraxial approximation, which holds in our case when manipulating collimated beams (propagating along the inclusion axis) using flat optical components. If the direction of wave propagation in angled with respect to the cylinder axis of the holes, the porous component may exhibit birefringence. However, in the following theoretical analysis and experimental demonstration, the incident wave is perpendicular to the input plane of the optical component, and, therefore, the wave travels parallel to the cylinder axis inside the component. Therefore, in this arrangement, the optical components are not polarization sensitive. Moreover, if in place of arrays of cylindrical inclusions produced by laser cutting one would use arrays of cubic or circular inclusions produced by 3D printing, the resultant porous materials will be nonbirefringent regardless of the direction of the propagating wave. In general, propagation through media with aligned cylindrical inclusions must be considered in the framework of anisotropic materials with distinct transverse and longitudinal values of the effective dielectric permittivity tensor.
In the limit of high porosity $f \to 1$ (high air filling fraction by area), from Equation (6) we can derive the following asymptotic expressions for the complex effective RI of the medium (see Section S1 in the Supporting Information for details)

$$n_{\text{eff}} = \text{Re} \left( \sqrt{E_{\text{eff}}} \right) = n_a + (1 - f) \frac{n_m - n_a}{n_m^2 + n_a^2} n_a$$  \hspace{0.5cm} (8)

$$\alpha_{\text{eff}} = \frac{4\pi}{\lambda} \text{Im} \left( \sqrt{E_{\text{eff}}} \right) = \alpha_m \frac{4(1 - f) n_m n_a}{(n_m^2 + n_a^2)^2}$$  \hspace{0.5cm} (9)

As we can see from Equation (9), effective material losses can be reduced by using high fraction of the loss-less inclusions, as well as by choosing high RI contrast between the host material and that of the subwavelength inclusions. The latter is easy to understand by looking at the field distribution in the porous materials (see Figure 1 (inset in (b)). There we observe a strong presence of the electric field in the low-loss material (hole). This is due to the continuity of the normal component of the displacement field across the interface, thus resulting in the electric field enhancement in the air hole compared to that in the host material $E_a \sim E_m \varepsilon_m / \varepsilon_a$. As we will see in what follows, this is an enhanced field presence in the low-loss subwavelength inclusions that allow designing porous optical components with lower losses compared to their solid counterparts with otherwise identical characteristics.

In Figure 1b,c, we compare the predictions of the Bruggeman model for the complex effective RI of the structured material with the numerical results obtained using the guided wave module of the finite element method (FEM) COMSOL Multiphysics software. In our simulation, we used a hexagonal periodic lattice (interhole spacing) of $\Lambda = 952 \mu m$ with periodic boundary condition at the boundaries of the hexagonal cell (Figure 1a), while varying the filling fraction and the frequency of the guided THz wave. Host material complex RI was taken as $n_m = n_m + i\kappa_m = 1.61 + 0.0085i$ which corresponds to that of a PMMA (acrylic) at 150 GHz—a material that is typically used in laser cutting. This value, which is consistent with the literature, was experimentally found with a THz continuous-wave (CW) spectroscopy system using a solid PMMA discussed later in the section. In Figure 1b,c we present the dependence of the real and imaginary parts of the effective RI as a function of the filling fraction and THz frequency. As expected, we observe that the Bruggeman model (dotted line) becomes equivalent to the numerical calculations in the limit of low and high filling fractions, and long wavelengths (low frequencies) at which the effective medium approximation is justified. In fact, at low frequencies, predictions of the Bruggeman model for the complex value of the material effective RI match well those of full simulations for any values of the filling fraction, which is mostly due to the relatively small RI contrast between PMMA and air. Interestingly, in Section S2 in the Supporting Information, we show that the numerical results can be fitted extremely well at all frequencies and filling fractions using a modified three component Bruggeman model.

The validity of the Bruggeman model was also verified experimentally by fabricating a series of porous plates with different air filling fractions (different hole diameters). We first experimented with additive manufacturing techniques, namely, fused deposition modeling (FDM) and stereolithography (SLA) for the fabrication of porous components. While FDM is a single step fabrication method that requires no post-processing, it has a limited resolution mainly dictated by the extruder nozzle size (in our case $=0.4 \text{ mm}$). On the other hand, SLA can achieve much higher nominal resolutions (as low as 50 $\mu m$), however it requires a postprocessing step of removal of the unreacted photosensitive resin from the narrow cylindrical channels, which frequently results in clogging and structural deformations. Although these fabrication methods could work relatively well for millimeter-wave applications (for which the pores can be large), for application in THz, further perfecting of the FMD and SLA techniques is required to reduce the pore size and consistent feature definition. For completeness, we mention that much precise techniques such as photolithography can also be used for the fabrication of porous components via stacking of the individual plates, however such techniques are generally expensive and not easily scalable when thick 3D-structured optical components are needed.

In this work, we report using laser cutting to construct our porous components. This technology is becoming widely available thanks to its relatively low cost. It is highly precise (beam spot size $=100 \mu m$, positional accuracy $=20 \mu m$), can produce thick optical components, and even has rudimentary 3D capabilities such as cutting holes at variable angles to the normal of a surface. For laser cutters, the precision of the fabrication depends on many factors such as the power of the laser, the repetition rate, the beam spot size, the cutting speed, the number of passes, and so on. Furthermore, the mechanical and thermal properties of the material are also important parameters. In principle, the laser cutting technology can work on a variety of materials, such as polymers, woods, glasses, metals, etc.

To fabricate our porous components, we used a commercially available Trotec Speedy 300 laser engraver, which consists of a CO2 laser with a maximum power of 120 W. We chose a 5.75 mm thick PMMA plate (acrylic), which is a standard material for the laser cutting. The plates featured a hexagonal pattern of holes with the period of $\Lambda = 952 \mu m$ (similar to the numerical simulations) and variable hole diameters (Figure 1d). As we will see in the following experimental demonstrations, the imperfections observed in the fabricated hole shapes and positions have little impact on the transmission properties, especially at lower frequencies. Moreover, when setting appropriate cutting laser parameters, the transmission characteristics are highly reproducible from sample to sample. The reproducibility and the robustness of the fabrication techniques are mainly due to the fact that the imperfections are deeply subwavelength, while even the imperfections themselves are reproducible from sample to sample due to high accuracy of the computer-controlled positioning system, as well as high quality of the laser used in the cutting process. To accurately determine the resulting filling fraction, we compared the weight of the porous plates with one without any hole.

The complex RI of the plates was then measured using a THz-CW spectroscopy system. A parallel THz beam was sent perpendicularly to the sample and then focused and detected using a fiber-coupled photomixer (more details about this system can be found in the work by Ma et al.27). We measured the transmitted THz amplitude and phase between 130
and 170 GHz, from which we obtained the real and imaginary parts of the RI using a measurement without any sample (reference) and the iterative algorithm described in the work by Guerboukha et al.\textsuperscript{[1]} We then averaged the RI in the spectral region to increase the accuracy of the measurement. In Figure 1e,f, we compare the experimental results with the Bruggeman model and the numerical simulation at 150 GHz. As expected, we observe that the effective complex RI of the plates follows the numerical simulations quite well.

In the following sections, we use the theoretical and experimental results presented so far for the complex RI of porous materials to design and fabricate a planar lens (Section 3) and an OAM phase plate (Section 4). These planar components were then characterized using THz imaging with a modified THz time-domain spectroscopy system, which allowed to map both the amplitude and the phase of the THz radiation at different frequencies.

3. Planar Porous Lenses

We now consider in more details the design of porous planar lenses and we compare their performance to their solid counterparts. We will discover that porous lenses can be designed to have much lower transmission losses than their all-solid analogues, while having otherwise identical optical properties.

For our comparison purposes, we start by designing a solid planoconvex lens with a radially symmetric thickness profile \( h(r) \) as shown in Figure 2a. We assume that the lens is made of a material with real part of the RI \( n_m \) and power absorption coefficient \( \alpha_m \). The lens radius is taken as \( R \) and the lens front focal length as \( F \). Assuming an incident planar wavefront, the lens thickness profile can be found by noting that the optical distance from any point on the lens input surface to the lens focal point should be the same (Fermat’s principle). We therefore write for two rays, one entering at the lens periphery \( R \) and the other entering at a distance \( r \) from the lens center

\[
\begin{align*}
    h(r) & = h(R) + \frac{n_a \sqrt{R^2 - r^2}}{2F} \\
    h(R) & = h_m + \frac{n_a R}{2} - \frac{n_a R}{2F} \\
    h_m & = n_m - n_a
\end{align*}
\]

Assuming that the lens radius and the lens thickness are much smaller than the lens focal distance \( h(R,F) \), \( F \ll R \), the equation above can be simplified to give the following lens thickness profile

\[
    h(r) = h(R) + \frac{n_a R^2 - r^2}{2F}
\]

Figure 2. a) Schematic of the planoconvex solid lens and b) of the porous lens with two optical rays. c) Analytical ratio of the transmission coefficients through the porous and solid lens as a function of the material refractive index for different \( \Gamma = (\alpha_m R^2)/2F \) as described by Equation (22). When the material has very high losses (\( \Gamma \gg 1 \)), the ratio tends to the asymptotic function defined in Equation (23) (black dotted line). d–f) Numerical ratio of the transmission coefficients through the porous and solid lens as a function of the \( n_m \) and \( \alpha_m \) for a lens with a focal length of 80 mm and a radius of 18 mm for d) 5.75 mm, e) 10 mm, and f) 20 mm. The gray areas correspond to regions where it is impossible to fabricate a porous lens with the given thickness, i.e., the condition at Equation (16) is not respected.
We can now estimate the total power transmission coefficient through the lens by integrating the transmission coefficients of the individual rays across the lens surface

$$T_{\text{lens}} = \frac{1}{\pi R^2} \int_0^\pi \int_0^{2\pi} \exp[-\alpha_s h(r)] \frac{1-\exp(-\xi)}{\xi} \, d\xi, \quad \xi = \frac{n_0 - n_s}{n_0 - n_a} \frac{R^2 - r^2}{2F}$$

where \( \xi = \frac{n_0 - n_s}{n_0 - n_a} \frac{R^2 - r^2}{2F} \), and \( T_{\text{solid}} \) is a monotonically decreasing function of \( \xi \), parameter. We note that the maximal transmission through the solid lens is achieved when the lens thickness goes to zero at the lens periphery \( h(R) = 0 \). In this case, we write

$$\text{max}(T_{\text{lens}}) = \frac{1-\exp(-\xi)}{\xi}$$

We now design a planar porous lens and compare its losses to the solid one. In the porous lens, the thickness \( h \) of the plate is fixed, while the local RI varies according to the radial profile \( n(r) \), where \( n_a < n(r) < n_m \). The lens radius is taken as \( R \) and the lens front focal distance as \( F \), which are the same as for the solid lens. As above, assuming a planar wavefront incident on the planar lens, the lens RI profile can be found by equating the optical paths for the two rays entering at the lens periphery \( P \) and at a distance \( r \) from the center (Figure 2b)

$$h \cdot n(r) + n_s \sqrt{r^2 + F^2} = h \cdot n(R) + n_s \sqrt{R^2 + F^2}$$

Assuming that \( h(r)F \ll R \), we can further simplify this equation to give the following lens RI profile

$$n(r) = n(R) + \frac{n_s}{h} R^2 - r^2$$

Furthermore, since \( n_a < n(R) < n_m \), there is a restriction on the minimal thickness of the porous plate for a given set of \( F \) and \( R \). Indeed, this minimal lens thickness is given by

$$h = \frac{n_a}{n(0) - n(R)} \left[ \sqrt{R^2 + F^2} - F \right] \Rightarrow h_{\text{min}} = \frac{n_a}{n_m - n_a} \frac{R^2}{2F}$$

We now turn to calculating the losses of the porous lens. Remembering that lens RI variation is enabled by controlling the local porosity of the material, then using Bruggeman approximation for the real part of the material RI (Equation (8)), we get the following expression for the porosity profile

$$n(r) \approx n_s + \left[1 - f(r)\right] \frac{n_m - n_s}{n_m - n_a} R^2 - r^2$$

which allows us to find the corresponding air filling fraction profile

$$f(r) = 1 - \frac{1}{\frac{n_a}{n_s} + 4n_a^2 n_m (n_m - n_a)} \left[n(R) - n_s + \frac{n_a}{h} \frac{R^2 - r^2}{2F} \right]$$

Using now the Bruggeman approximation for the imaginary part (Equation (9)), we can now establish the absorption profile of the porous lens

$$\alpha(r) = \alpha_a \left[1 - f(r)\right] \frac{4n_a^2 n_m}{n_m^2 - n_a^2} \left[n(R) - n_s + \frac{n_a}{h} \frac{R^2 - r^2}{2F} \right]$$

As previously, we can now estimate the total power transmission coefficient through the porous lens of profile \( n(r) \) by the integral

$$T_{\text{porous}} = \frac{1}{\pi R^2} \int_0^\pi \int_0^{2\pi} \exp[-\alpha_s h(r)] \frac{1-\exp(-\xi)}{\xi} \, d\xi, \quad \xi = \frac{n_0 - n_s}{n_0 - n_a} \frac{R^2 - r^2}{2F}$$

where \( \xi = \frac{n_0 - n_s}{n_0 - n_a} \frac{R^2 - r^2}{2F} \), and \( T_{\text{solid}} \) is a monotonically decreasing function of \( \xi \), parameter. The maximal transmission through the porous lens is achieved when the lens RI at the lens periphery equals that of air \( n(R) = n_a \)

$$\text{max}(T_{\text{porous}}) = \frac{1-\exp(-\xi)}{\xi}$$

By comparing Equation (13) of the solid lens and Equation (21) of the porous lens, we conclude that the transmission coefficient through the optimally designed porous lens is always higher than the transmission coefficient through the corresponding solid lens of identical radius and focal distance. This follows directly from the observation that \( \xi > \xi \), for any value of the lens material refractive index \( n_m \) and that of the surrounding medium \( n_a \) as long as \( n_a > n_m \). The effect is more pronounced for higher refractive index contrast between the two.

Interestingly, from Equations (13) and (21), we can now obtain an analytical expression for the ratio between the transmission coefficients through a porous lens and a solid lens

$$\max\left(\frac{T_{\text{porous}}}{T_{\text{solid}}}\right) = \frac{n_m - n_a}{4n_a^2 n_m (n_m - n_a)} \left[1 - \exp\left(-\Gamma \frac{n_a}{n_m - n_a}\right)\right]$$

with \( \Gamma = \frac{n_a \ R^2}{2F} \). In Figure 2c, we plot this ratio as a function of \( n_m \) for different values of \( \Gamma \). As it can be observed, the ratio is greater than 1 for all values of \( n_m \) and \( \Gamma \), indicating that the porous lens outperforms the solid one. In particular, in the limit of high absorption losses \( \Gamma \gg 1 \), one can calculate the following asymptotic result, which is represented by the dotted line in Figure 2c

$$\lim_{\Gamma \rightarrow \infty} \left[\frac{\max(T_{\text{porous}})}{\max(T_{\text{solid}})}\right] = \frac{n_m - n_a}{4n_a^2 n_m (n_m - n_a)}$$

We note that the analytical ratio obtained in Equation (22) is valid in the limit of high porosities \( f \rightarrow 1 \). This is due to the use of the approximation of the Bruggeman model in Equations (8) and (9). In the following, we numerically compute the ratio of...
Obtain the loss profile $h(R)$ is obtained with Equation (10). To find the corresponding distribution of the air-filling fraction $f(r)$, we placed the imaginary part of Equation (6) with thus found $f(r)$, we find the complex value of the lens RI profile, and from Equation (7), the loss profile $\alpha(r)$.

In Figure 2d–f, we map the ratio $\frac{T_{\text{trans}}^{\text{lens porous}}}{T_{\text{solid}}^{\text{lens}}}$ obtained numerically with Equation (24) as a function of the material refractive index $n_m$ and the absorption loss $\alpha_m$ for a lens with a focal length of 80 mm, a radius of 18 mm, and thicknesses of 5.75, 10, and 20 mm. The numerical results show that this ratio is greater than 1 for all values of $n_m$ and $\alpha_m$, indicating that the porous lens always outperforms (in terms of losses) the solid planoconvex lens with comparable optical properties. In particular, the overall ratio increases with the thickness of the plate, since thicker plates can be fabricated with plates with more porosity. In Figure 2d–f, the hatched areas indicate the regions where it is impossible to make a lens with such a value of the thickness, i.e., $h$ is smaller than the minimal thickness stated in Equation (16).

Using laser drilling in a PMMA plate of thickness $h = 5.75$ mm, we then designed and fabricated a planar lens with $F = 80$ mm and $R = 18$ mm (Figure 3a,b). The porous lens was realized by drilling holes of various diameters at the vertices of a hexagonal lattice with period $\Lambda = 952 \mu$m (identical to the FEM simulations). More precisely, for every vertex of a lattice, we would first find its distance $r_i$ to the lens center. Then, using Equation (15), we would calculate the desired refractive index at that position. Finally, using the FEM data for the porous material refractive index as a function of the material porosity at 150 GHz (Figure 1e), we would find the target filling fraction and the corresponding hole diameter $D_i$ to be placed at the vertex location.

When designing a porous lens, one can, in principal, select any value of the refractive index $n(R)$ at the lens edge in Equation (15). However, to design a porous lens with minimal losses, the value of $n(R)$ has to be set as low as possible (see Equation (19)). In practice, $n(R)$ cannot be equal to $n_0 = 1$ simply because the filling fraction for circular holes in a hexagonal lattice is limited to $\approx 0.91$, while that in a square lattice is limited to $\approx 0.79$. The maximal filling fraction is further reduced to ensure proper fabrication and mechanical stability. Experimentally, we found that a maximal value of the filling fraction for a hexagonal lattice is $\approx 0.7$, which gives $n(R) = 1.2$ for a PMMA substrate. After fixing $n(R)$, the RI monotonically increases following Equation (15) to reach its maximal value in the center of the lens $n(0) = n(R) + (n_0 - n(R)) \cdot R^2/2F$. Since this value cannot exceed the bulk refractive index of the PMMA material $n_m$, this sets an upper limit on the diameter and the focal length of the lens as expressed by Equation (16). Typically, for shorter focal lengths, the diameter of the porous lens must be reduced. One can also increase the thickness of the planar lens to increase the overall porosity and reduce the losses.

To characterize the planar lens, we imaged the THz beam at various focal positions using a modified THz time-domain spectroscopy system with a movable detector (Figure 3c). This imaging system allowed us to obtain both amplitude and phase of the THz beam at different frequencies (for more details about this system, see the work by Guerboukha et al.\[35\]). We first imaged the reference beam incident on the planar lens. The intensity at 150 GHz is shown in Figure 4a. We measured a full width at half maximum (FWHM) of $\approx 45$ mm, which is larger than the porous lens diameter. This ensures that the incident beam has a uniform phase distribution. Then, we placed the planar lens and imaged the THz beam in the $yz$ plane (Figure 4b). We observed that the beam was correctly focused at the focal distance $F$. In Figure 4c, we imaged the THz beam in the $xy$ plane at various $z$ positions, which allowed us to see the focusing behavior of the beam. In Figure 4d, we performed a detailed analysis of the focused beam at several wavelengths by...
fitting Gaussian curves in the spectral band from 75 to 300 GHz. By fitting the FWHM measured in the focal plane, we obtain

$$\text{FWHM} = 1.24 \frac{\lambda F}{D}$$  \hspace{1cm} (25)$$

which is represented by the red line in Figure 4e. The obtained fit is close to the Rayleigh criterion defining diffraction-limited focusing $\delta x = 1.22 \frac{\lambda F}{D}$.

Even though the lens has been designed for the frequency of 150 GHz, we observe experimentally that its operational bandwidth extends from 100 to 300 GHz (see Figure 4d,e). This is in part due to the fact that the refractive index of PMMA is relatively constant in this spectral region. In the inset of Figure 4e, we plot the lens efficiency as a function of frequency obtained by dividing the detected power in the focal plane of a porous lens (Figure 4c) by the power of the incident collimated THz beam (Figure 4a). The powers were calculated directly from the acquired images by summing the pixel values across the total area of the lens. For the design frequency of 150 GHz, we obtain a power transmission around $\approx 70\%$, which becomes even larger at lower frequencies.

When the frequency is increased well above the design frequency, the THz wavelength becomes comparable or smaller than the pore size, which leads to the breakdown of the effective medium theory and higher scattering losses. On the other hand, when decreasing the THz frequency, the effective medium theory (Bruggeman model) becomes even more valid (as can be clearly observed in Figure 1b,c) and the designed lens operates as intended. In principle, the porous lens can be designed for any spectral region, by properly scaling the dimensions of the pores and the unit cell.

Furthermore, from Figure 4d, we observe geometrical aberrations in the focused beam at high frequencies. For example,
at 250 GHz, we notice nonsymmetric side lobes of non-negligible amplitudes. Since these aberrations appear only at higher frequencies, we believe that they are due to the imperfections in the fabricated pore shapes and positions (see Figure 1d). However, a more detailed study is needed to evaluate quantitatively the frequency dependent aberrations of porous optical components and develop strategies to mitigate them.

4. Planar Porous Orbital Angular Momentum Phase Plates

In this section, using a methodology similar to previously, we present the design and fabrication of porous OAM phase plates. The OAM is a property of light which presents itself as an electric field with an azimuthal phase term $E \sim e^{im\phi}$, characterized by the topological charge $m$ that indicates the number of helices in a full rotation. The full expression can be obtained from the Laguerre–Gaussian decomposition with a helical phase and a donut-shaped amplitude with a singularity in the center.[36]

A simple method of generating OAM states from a planar wave is to use a spiral phase plate with a thickness that varies as a function of the angle

$$h(\theta) = h(0) \pi \frac{m\theta}{2\lambda}$$

where $\lambda$ is the wavelength and $h(0)$ is the thickness at the polar origin (Figure 5a). Our approach will be to fix the

![Figure 5. Schematic of a) the solid OAM phase plate and b) porous phase plate, where an incident planar wave is converted into a vortex beam. Transmission ratios of the OAM phase plate computed numerically with Equation (37) as a function of $n_m$ and $\alpha_m$ at a frequency of 140 GHz for $m = 1$ with c) $h = 5.75$ mm, d) $h = 10$ mm, e) $h = 20$ mm, and for $m = 2$ with f) $h = 5.75$ mm, g) $h = 10$ mm, and h) $h = 20$ mm. The gray areas correspond to regions where it is impossible to fabricate a porous OAM phase plate, i.e., the condition at Equation (28) is not respected.](image-url)
thickness $h$ of the plate and vary the local RI as a function of the angle

$$n(\theta) - n(0) = \frac{\lambda}{h} \frac{m \theta}{2\pi}$$

(27)

where $n(\theta)$ is the RI at the polar origin (Figure 5b). Since $n_s < n(\theta) < n_m$, the minimal thickness of an OAM plate that can realize a topological charge $m$ is

$$h_{\text{OAM}} = \frac{m \theta}{2\pi} \frac{\lambda}{n_m - n_s}$$

(28)

where we must require that $n(0) = n_s$ and $n(2\pi) = n_m$.

As it can be seen from Equations (26) and (27), an OAM phase profile for the topological charge $m$ is designed for a specific wavelength. Moreover, in the absence of dispersion, the phase profile after the phase plate depends linearly on frequency (see Equation (1)). Thus, at shorter wavelengths, the topological charge is no longer equal to the designed one and can even take noninteger values, meaning that the phase is making more than $m$ turns around the origin. In particular, when the wavelength is half the designed wavelength, the topological charge of the OAM state at that wavelength becomes $2m$. Therefore, by judiciously designing the OAM phase plate, one could use the same OAM phase plate to obtain $m = 1$ at $\lambda$ and $m = 2$ at $\lambda/2$, which may be useful in multiplexing multiple OAM states in a broadband spectrum.

In Section S3 in the Supporting Information, using a similar methodology as in the case of the porous lens, we demonstrate analytically that the porous OAM phase plate always outperforms the all-solid phase plate in terms of losses. Indeed, we arrive at the following equation for the power ratio between the transmission through a porous and a solid OAM phase plate

$$\frac{\max(T_{\text{OAM porous}})}{\max(T_{\text{OAM solid}})} = \frac{n_m - n_s}{4n_s^2 (n_m - n_s)} \left[ 1 - \exp\left(-\frac{4n_s n_m}{n_m - n_s} - 1\right) \right]$$

(29)

Remarkably, this expression is similar to the one calculated previously for the lens, where the only difference is that $\Gamma = \alpha_m \lambda$. Therefore, the graph in Figure 2c applies also for the OAM phase plate, where we can observe that the ratio is always greater than 1 and increases with $\Gamma$.

As previously, to avoid the use of the Bruggeman approximations for high porosities ($f \rightarrow 1$), we numerically compute the ratio of the power transmission through the porous and the solid OAM phase plate, assuming $h(0) = 0$ and $n(0) = 1$

$$\frac{T_{\text{OAM porous}}}{T_{\text{OAM solid}}} = \frac{\int_0^{2\pi} d\theta \exp(-\alpha(\theta) h)}{\int_0^{2\pi} d\theta \exp(-\alpha_m h(\theta))}$$

(30)

where $h(\theta)$ was obtained with Equation (26) and we use the Bruggeman model (Equation (6)) to obtain $\alpha(\theta)$, in a manner similar to previously.

In Figure 5c–h, we map the ratio $T_{\text{OAM porous}} / T_{\text{OAM solid}}$ obtained numerically with Equation (30) as a function of the material refractive index $n_m$ and the absorption losses $\alpha_m$. We select a frequency of 140 GHz, suited for our THz communications system,[37] and we compute the ratio for orders $m = 1, 2$ for different porous plate thicknesses 5.75, 10, and 20 mm. In all the cases, the porous plate outperforms the solid one, and we observe that the performance is increased with the order $m$ and the thickness $h$.

Using laser drilling in a PMMA plate, we then designed and fabricated an OAM phase plate of 50 mm diameter with a topological charge of $m = 1$ for a frequency of 140 GHz (Figure 6a,b). We first placed a square lattice with an interhole distance of 1100 $\mu$m and we calculated the angle $\theta$ of every hole center. This allowed us to map the refractive index profile using Equation (27). Using simulation results for 140 GHz, we then found the corresponding filling factor and diameter $D_i$. Using our THz time-domain spectroscopy system, we then imaged the THz beam at 140 GHz. The reference beam without the OAM phase plate shows a Gaussian-like amplitude distribution (Figure 6c) and a planar phase front (Figure 6d). When placing the porous OAM phase plate, the amplitude is transformed into a donut (Figure 6e) with a phase varying azimuthally from $-\pi$ to $+\pi$ (Figure 6f).

We compare our experimental results with the analytical expression of the Laguerre–Gaussian orthogonal mode set for the electric field, which is a standard definition used to describe OAM states.[38]

$$LG_m(r, \phi) = \frac{1}{w_0} \sqrt{\frac{2p!}{\pi (p + |m|)!}} \left( \frac{r}{w_0} \right)^{|m|} \exp\left(-\frac{r^2}{w_0^2}\right) L_p^{m+|m|}\left(\frac{2r}{w_0}\right) \exp(i\phi)$$

(31)

where $L_p^{m+|m|}$ is the generalized Laguerre polynomial obtained from the Laguerre polynomial $L_p$ with

$$L_p^{m+|m|}(x) = (-1)^{|m|} \frac{d^{|m|}}{dx^{|m|}} L_p(x)$$

(32)

In Equation (31), $w_0$ is the beam waist (FWHM/$\sqrt{2\ln 2}$), $m$ is the OAM topological charge, and $p$ is the number of radial nodes in the intensity profile. In Figure 6g,h, we present respectively the amplitude and phase of Equation (31) using $w_0 = 10$ mm, $m = 1$, and $p = 0$. Compared to the Laguerre–Gaussian mode, we observe angular variations in the amplitude in our experimental results. This is directly caused by the nonuniform loss distribution of the porous material (see Equation (S17) in the Supporting Information). As for the experimental phase, it agrees well with the theoretical expression, with a phase varying azimuthally from $-\pi$ to $+\pi$.

5. Conclusion

In conclusion, we have designed, fabricated, and characterized low-loss planar porous components for THz beamforming. Our approach uses low-RI subwavelength inclusions in a solid material to design optical components with smaller losses than their all-solid counterparts with otherwise identical optical properties. This is possible due to the fact that the electric field
is enhanced in the subwavelength air holes compared to that in the host material, which results in a faster decrease of the losses of the effective material compared to its refractive index when increasing porosity. Using the Bruggeman effective medium theory, we demonstrated the validity of this approach analytically and confirmed it with numerical simulations and experimental measurements. We then fabricated and characterized a planar porous lens and a planar porous OAM phase plate. We observed a diffraction-limited focusing using the planar lens, as well as a THz beam with a topological charge of 1 using the OAM phase plate. In principle, one could even combine both the lens and the OAM phase plate into a single porous component, since the first requires radial dependence of the RI profile while the second rather uses angular dependence. We believe that this work opens a new research direction aimed at using nominally high-loss materials with other interesting functionalities (for example high RI, high group velocity dispersion, etc.) to build novel low-loss THz components.

Supporting Information

Supporting Information is available from the Wiley Online Library or from the author.

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Conflict of Interest

The authors declare no conflict of interest.

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