Fast Compressionless Reconstruction for Terahertz Imaging

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***Abstract*—We introduce a novel fast compressionless reconstruction algorithm for THz imaging. Based on Fourier imaging and a broadband THz spectrum, we demonstrate image reconstruction with high resolution using only 10s of pixels. First, we develop a mathematical framework based on polar Fourier optics and we demonstrate four cases where the fast reconstruction is possible. Second, we experimentally measure two of these cases using a fiber-coupled THz-TDS system.**

I. INTRODUCTION

FOR many years, THz imaging has been applied to many different fields such as security, industrial, agricultural etc. However, some difficulties remain and prevent a generalized use of THz. Among those difficulties, we note the difficulty to produce highly sensitive multi-pixel detectors. In the meantime, terahertz time-domain spectroscopy (THz-TDS) is an established detection technique with high dynamic range. Both amplitude and phase of the THz electric field are temporally resolved, leading to spectral fingerprints identification on top of imaging. Most commonly, THz imaging is performed with fixed detector and emitter, by spatially moving the sample located at the focal point of two focusing optics. This can be problematic in many situations where the sample cannot be practically moved.

Moreover, an image with *N*x*N* pixels is acquired by moving the sample at N^2 positions. Already, compressive sensing theory was used to reduce the number of necessary points for image reconstruction [1] and to use a single-pixel fixed detector [2]. The main idea is to use the sparsity on a given basis to numerically reduce the number of points. In [1], the authors show that 12% of their 4096 points are needed to reconstruct a 64x64 pixel image at 200 GHz (492 pixels). In [2], they reconstruct an 32x32 image using between 29% and 59% of the total pixels at 100 GHz (between 297 and 604 pixels).

In this work, without the need to rely on compressive sensing theory, we demonstrate an imaging reconstruction technique that uses as little as 40 pixels to efficiently reconstruct an image. The fundamental idea is to use the broadband nature of the pulse to construct the k-space by sampling along a 1D circle around the origin. The image is then reconstructed using an inverse Fourier transform. We begin by showing how the Fourier optics expressions are transformed when we pass from a spatial integration to a spectral integration. Our mathematical framework will lead us to two practical cases where the fast reconstruction is possible: a binary mask and a phase mask. Finally, we experimentally demonstrate the fast reconstruction in those two cases.

II. MATHEMATICAL FRAMEWORK

The Fourier optics theory states that an object $S(x,y,\nu)$ located at a focal distance F from a lens is Fourier-transformed:

$$U(\xi,\eta,\nu)=\frac{\nu}{jcF}\iint \mathrm{d}x\mathrm{d}y S(x,y,\nu)\exp\left[-\frac{j2\pi\nu}{cF}(x\xi+y\eta)\right]$$

where the spatial frequencies are related to the THz frequency ν by $k_\xi=\xi\nu/cF$ and $k_\eta=\eta\nu/cF$. Therefore, by raster scanning the Fourier plane, the object $S(x,y,\nu)$ can be then reconstructed using the usual inverse Fourier transform:

$$S(x,y,\nu)=\frac{jcF}{\nu}\iint \mathrm{d}\xi\mathrm{d}\eta\, U(\xi,\eta,\nu)\exp\left[-\frac{j2\pi\nu}{cF}(x\xi+y\eta)\right]$$

Owing to the broadband nature of THz-TDS measurement, a single point at position (ξ_0,η_0) will produce a line in k-space. Hence, by measuring along a circle of radius ρ_0 in the (ξ,η) plane, we can construct the k-space. Essentially, the Cartesian integration is transformed to a polar, and the integration over the radial coordinate is made through the THz spectrum:

$$S(r,\theta)=\iint \mathrm{d}\phi\nu\mathrm{d}\nu\left(\frac{j\rho_0^2}{cF\nu}\frac{U(\phi,\nu)}{U_0(\nu)}\right)\exp\left[-\frac{j2\pi\rho_0}{cF}r\nu\cos(\theta-\phi)\right]$$

In THz-TDS, a reference measurement $U_0(\nu)$ needs to be used in order to have a slowly varying phase. The choice of this reference depends on the nature of $S(\vec{r},\nu)$. We mathematically study four cases where the fast reconstruction is possible:

1. Frequency independent image: $S(\vec{r},\nu)=S(\vec{r})$
2. Space-frequency separable image: $S(\vec{r},\nu)=S(\vec{r})I(\nu)$
3. Space-frequency separable image with a uniform phase shift: $S(\vec{r},\nu)=S(\vec{r})I(\nu)\exp(i2\pi\nu\Delta/c)$
4. Space-frequency separable image with a spatially dependent phase shift $S(\vec{r},\nu)=S(\vec{r})I(\nu)\exp(i2\pi\nu\Delta(\vec{r})/c)$

In practice, case 2 and 4 are the most interesting. For example, case 2 can be the imaging of a metallic binary mask. In case 4, $\Delta(\vec{r})=nL$ is the optical path length (phase mask). Therefore, it could be applied to the determination of a defect in a polymer material. For these two cases, we develop specific reconstruction algorithms. For the binary mask, we show that the reference can be taken at the origin of the (ξ,η) plane such that $U_0(\nu)=(jcF/\nu)\cdot U(0,0)$. For the phase mask, for convergence considerations of the integrals, the reference should be taken as $U_0(\nu)=jcF\cdot U(0,0)$.

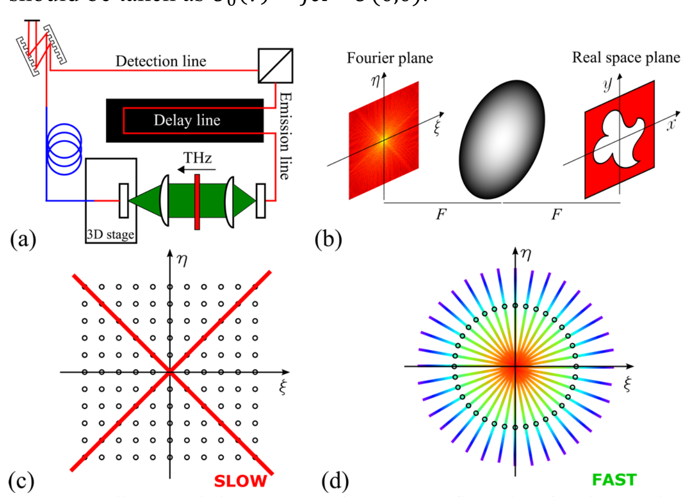

Fig.1. (a) Fiber-coupled THz-TDS. (b) Lens configuration for the Fourier transform, showing the real space plane and the Fourier plane located at a focal distance from the lens. (c) Standard slow raster-scanning method involving 2D sampling. (d) Our fast approach on a 1D circular sampling. The radial dimension is given by the THz spectral sampling.

III. Experimental Results

The experimental setup is depicted in Fig 1a. It is based on a fiber-coupled THz-TDS system. The THz emitter is fixed while the detector is mounted on a 3D stage for imaging. On the emission line, a lens acts as the Fourier transformer of a transmission object placed on its back focal plane (Fig. 1b).

First, we study the binary mask case. By using a metallic mask of the letter P at the focal plane, we measure the Fourier plane on a 100x100 mm grid with a 1 mm step resolution (10 000 pixels). We show in Fig. 2ab the absolute value and phase at a fixed frequency of 0.71 THz. The corresponding reconstructed image in Fig. 2e is calculated using the conventional iFFT algorithm. Using our fast scanning approach, we show in Fig. 2cd the constructed k-space using as little as 160 pixels in the (ξ, η) plane. Those points are sampled in a 20-mm radius around the origin at equal angle spacing. The reconstruction in Fig. 2f is performed by a brute-force integral in polar coordinates. Next, we compare the reconstructed image when using less points (between 10 and 80 pixels in Fig.1g to 1j). Using as little as 40 pixels, we can reconstruct the object.

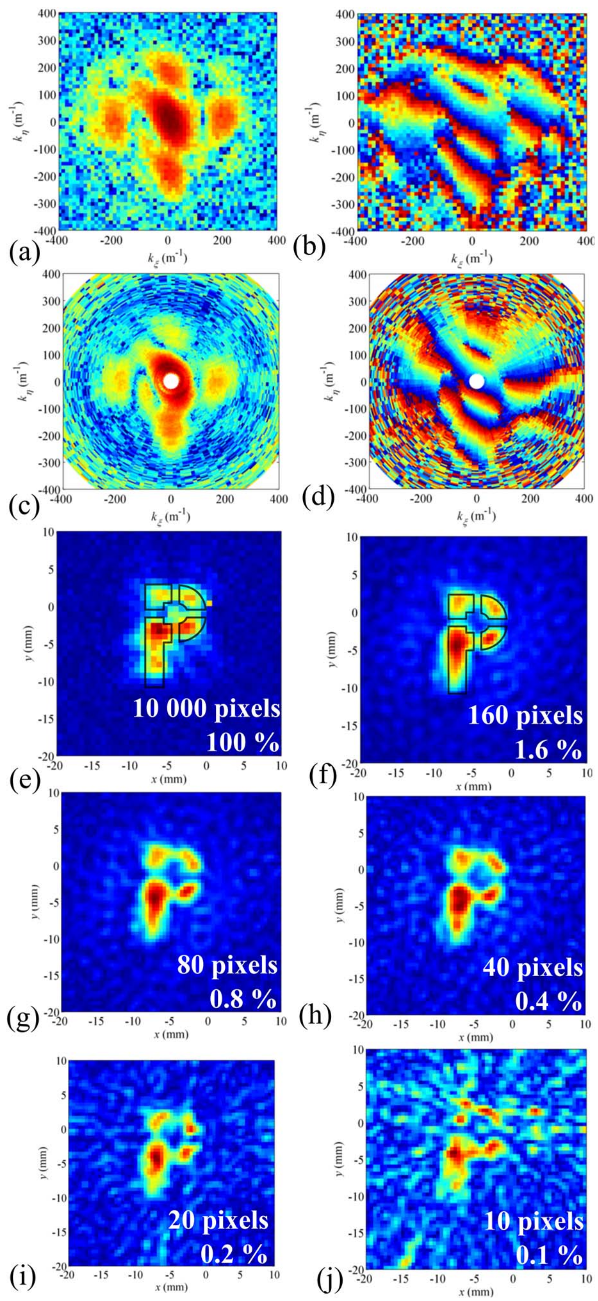

Fig.2. Efficient reconstruction of a binary mask. (a) Amplitude and (b) phase of the k-space at 0.71 THz. (c) Construction of the k-space amplitude and (d) phase using 160 pixels. (e) Single THz frequency reconstruction. (f) Efficient reconstruction using 160, (g) 80, (h) 40, (i) 20 and (j) 10 pixels.

Second, we experimentally study the phase mask case. Using a 3D printer, we fabricate a 2-mm thick resin sample with the letter H inscribed as a 100-µm deep scratch. As a reference, we take a similar 2-mm thick sample without the inscription. In the Fourier plane, the phase will be modified by the letter H. Thus, the absolute value of the constructed k-space is mostly Gaussian (Fig. 3a) while the information about the scratch is encoded in the phase (Fig. 3b). In the reconstructions (Fig. 3cef), we see both the THz Gaussian beam and the H letter.

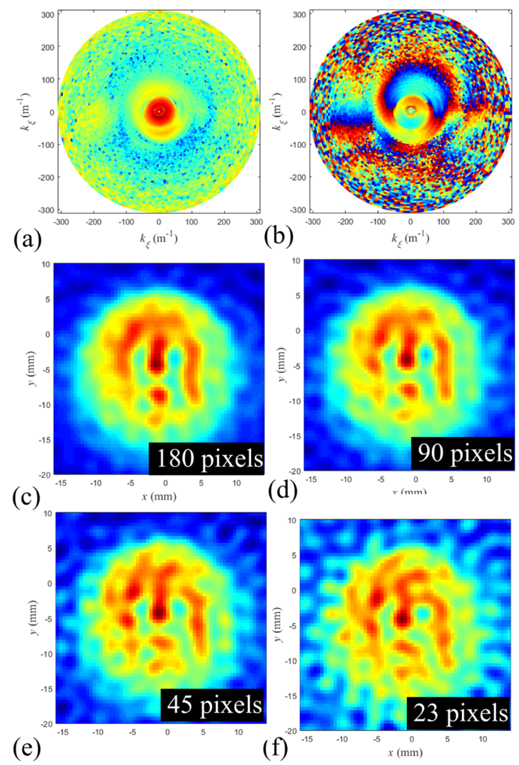

Fig.3. Efficient reconstruction of a phase mask. (a) Amplitude and (b) phase of the constructed k-space using 180 pixels and (c) corresponding reconstruction. (d) Reconstruction using 90, (e) 45 pixels and (f) 23 pixels.

IV. Summary

In summary, we have theoretically and experimentally demonstrated a THz compressionless reconstruction technique that uses very few pixels in the Fourier plane. The points are sampled along a circle and the object is reconstructed using a polar inverse Fourier transform. We then showed two practical cases, where the fast reconstruction is applicable: a binary mask and a phase mask.

References

[1]. W. L. Chan, M. L. Moravec, R. G. Baraniuk, and D. M. Mittleman, "Terahertz imaging with compressed sensing and phase retrieval," *Optics Letters*, vol. 33, pp. 974-976, 2008.

[2]. W. L. Chan, K. Charan, D. Takhar, K. F. Kelly, R. Baraniuk and D. M. Mittleman, "A singl-pixel terahertz imaging system based on compressed sensing," *Applied Physics Letters*, vol. 93, pp. 121105, 2008.