Rigid vibrations of a photonic crystal and induced interband transitions

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We investigate the behavior of electromagnetic states associated with photonic crystals, which are undergoing rigid time-dependent translations in position space. It is shown, quite generally, that the Bloch wave vector q remains a conserved quantity and that an analogue of Bloch's theorem for a time-dependent solution of the states can be formulated. Special attention is focussed on time-dependent translations involving harmonic rigid vibrations of the photonic crystal. Under these conditions it is shown how, and to what extent, inter-band transitions can be induced between the various bands in a photonic crystal in a microwave regime. In particular, a characteristic resonance transition time can be derived, which scales inversely with the amplitude of vibration and interband frequency. Finally, it is argued that given all parameters other than Bloch wave vector fixed, an interband transition time is minimized if the transition is made at a Bragg plane.

I. INTRODUCTION

The idea of using periodic dielectric materials to alter the dispersion relation of photons[1–5] has received widespread interest and consideration because of numerous potential applications.[6] It has been shown by several authors[7–9] that passive elements such as waveguide bends, channel drop filters, mirror surfaces, etc. can be substantially improved if constructed on the basis of photonic crystals. Recently, a strong interest has developed for the incorporation of nonlinear materials into photonic crystals. Investigations in the framework of field dependent dielectric media have led to several suggestions[10–12] on the possibility of constructing active elements such as optical switches and on the realization of dynamical effects such as second harmonic generation and induced interband transitions in photonic crystals.

The idea of this paper is to demonstrate the possibility of inducing interband transitions in photonic crystals using ordinary, linear field-independent media. To introduce a coupling between the electromagnetic states of a photonic crystal we employ rigid mechanical vibrations of the crystal with a driving frequency Ω and an amplitude Δ. It will be shown that in this setting, tuning the driving frequency to the frequency of the interband transition leads to coupling of the modes and an interband transition time that is inversely proportional to both the amplitude of vibration and the interband frequency. Experimentally, since the driving frequency should be comparable to the frequencies of the photonic modes, this method of inducing interband transitions should be most relevant to the microwave region.

The outline of the paper is as follows. In Sec. II, we describe a general approach for solving Maxwell's equations for general rigid time-dependent translations of a photonic crystal. Section III, deals with setting up the correct boundary conditions for the fields on the moving interface between two dielectrics. In Sec. IV, we address a computational scheme for obtaining the time-dependent population amplitudes of the electromagnetic modes. In Sec. V, harmonic vibrations of a photonic crystal are considered and resonant-mode coupling and photonic interband transitions are demonstrated. The interband transition time and its dependence upon various system parameters is discussed in Sec. VI. Finally, in Sec. VII, we make some concluding remarks. In the Appendix, we provide a detailed derivation of a modified Bloch theorem for a photonic crystal undergoing rigid time-dependent vibrations.

II. RIGID TRANSLATIONS OF A PHOTONIC CRYSTAL AND THE MASTER EQUATION

We begin by deriving the time-dependent field equations for the case of a translated photonic crystal. Starting with Maxwell's equations for a nonmagnetic material with a time- and position-dependent dielectric constant we have:

$$\begin{aligned} \nabla\times H(x,t) &= \frac{\partial[\epsilon(x,t)E(x,t)]}{\partial ct}, \\ \nabla\times E(x,t) &= -\frac{\partial H(x,t)}{\partial ct}. \end{aligned} \tag{1}$$

The equation for the magnetic field can be analyzed further by manipulating the right-hand side:

$$\begin{aligned} \nabla\times H(x,t) &= \frac{\partial\epsilon(x,t)}{\partial ct}E(x,t)+\epsilon(x,t)\frac{\partial E(x,t)}{\partial ct}, \\ \frac{1}{\epsilon(x,t)}\nabla\times H(x,t) &= \frac{\frac{\partial\epsilon(x,t)}{\partial ct}}{\epsilon(x,t)}E(x,t)+\frac{\partial E(x,t)}{\partial ct}, \\ \nabla\times\left[\frac{1}{\epsilon(x,t)}\nabla\times H(x,t)\right] &= \nabla\times\left[\frac{\frac{\partial\epsilon(x,t)}{\partial ct}}{\epsilon(x,t)}E(x,t)\right] \\ &\quad +\nabla\times\frac{\partial E(x,t)}{\partial ct}. \end{aligned} \tag{2}$$

Substitution of the second of Maxwell's equations from Eq. (1) into the last equation of Eq. (2) gives a time dependent version of the ordinary[5] master equation:

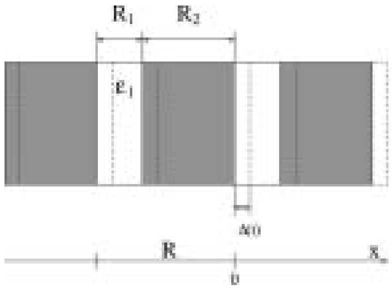

FIG. 1. 1D photonic crystal with periodicity R rigidly translated with a displacement $\Delta(t)$.

$$\nabla\times\left[\frac{1}{\epsilon(x,t)}\nabla\times H(x,t)\right]=-\frac{1}{c^2}\frac{\partial^2 H(x,t)}{\partial t^2}+\nabla\times\left[\frac{\frac{\partial\epsilon(x,t)}{\partial ct}}{\epsilon(x,t)}E(x,t)\right]. \tag{3}$$

Let us now consider a photonic crystal translated with a displacement $\Delta(t)$ as illustrated in Fig. 1, where R is the periodicity of the photonic crystal and R_1, R_2 are the widths of the slabs with dielectric constants ϵ_1 and ϵ_2 consequently.

For simplicity we focus on a one-dimensional (1D) photonic crystal but our analysis is valid in general. At a fixed point in space, the dielectric constant is typically a discontinuous function of time. We notice also that as soon as $x\neq\Delta(t)+Rl$, $x\neq\Delta(t)+R_1+Rl$ (boundaries between the slabs of different ϵ) where l is any integer, the term

$$\nabla\times\left[\frac{\frac{\partial\epsilon(x,t)}{\partial ct}}{\epsilon(x,t)}E(x,t)\right]=0. \tag{4}$$

Thus, the time dependent master equation reduces to a stationary photonic crystal master equation (i.e., the master equation for a photonic crystal at rest)

$$\nabla\times\left[\frac{1}{\epsilon[x-\Delta(t)]}\nabla\times H(x,t)\right]=-\frac{1}{c^2}\frac{\partial^2 H(x,t)}{\partial t^2}, \tag{5}$$

where $x\neq\Delta(t)+Rl$, $x\neq\Delta(t)+R_1+Rl$. We must now consider what happens with the fields on the moving boundary between two dielectrics.

III. RIGID TRANSLATIONS AND BOUNDARY CONDITIONS ON THE INTERFACE OF TWO MOVING DIELECTRICS

Let us consider a one dimensional interface between two dielectrics as shown in Fig. 2.

Maxwell's equations for each dielectric become

$$-\frac{\partial H(x,t)}{\partial x}=\frac{\partial\epsilon(x,t)}{\partial ct}E(x,t)+\epsilon(x,t)\frac{\partial E(x,t)}{\partial ct},$$
$$\frac{\partial E(x,t)}{\partial x}=-\frac{\partial H(x,t)}{\partial ct}. \tag{6}$$

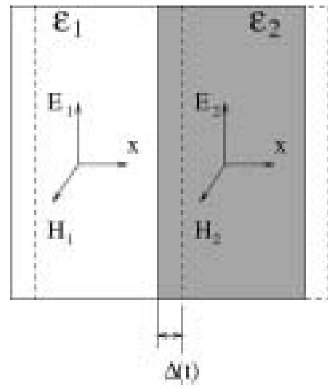

FIG. 2. 1D interface between two dielectrics displaced by $\Delta(t)$ at time t.

A dielectric medium which is a function of space and time can be conveniently expressed as

$$\epsilon(x,t)=\epsilon_1+(\epsilon_2-\epsilon_1)\theta[x-\Delta(t)], \tag{7}$$

where $\theta(x)$ is a standard step function. Thus, the time derivative of the dielectric media can be derived from this form of $\epsilon(x,t)$ and becomes

$$\frac{\partial\epsilon(x,t)}{\partial ct}=-(\epsilon_2-\epsilon_1)\delta[x-\Delta(t)]\frac{\dot{\Delta}(t)}{c}. \tag{8}$$

Maxwell's equations with the discontinuous term at $x=\Delta(t)$ can be satisfied if we assume discontinuous electric and magnetic fields through the interface. Thus, choosing

$$E(x,t)=E_1+(E_2-E_1)\theta[x-\Delta(t)],$$
$$H(x,t)=H_1+(H_2-H_1)\theta[x-\Delta(t)] \tag{9}$$

in the vicinity of the interface for the space and time derivatives of these fields we obtain

$$\frac{\partial E(x,t)}{\partial ct}=-(E_2-E_1)\delta[x-\Delta(t)]\frac{\dot{\Delta}(t)}{c},$$
$$\frac{\partial E(x,t)}{\partial x}=(E_2-E_1)\delta[x-\Delta(t)],$$
$$\frac{\partial H(x,t)}{\partial ct}=-(H_2-H_1)\delta[x-\Delta(t)]\frac{\dot{\Delta}(t)}{c},$$
$$\frac{\partial H(x,t)}{\partial x}=(H_2-H_1)\delta[x-\Delta(t)]. \tag{10}$$

Substitution of these derivatives into Maxwell's equations leads to the following equations for the boundary conditions:

$$-(H_2-H_1)\,\delta[x-\Delta(t)]=-(E_2-E_1)\frac{\dot{\Delta}(t)}{c}\,\delta[x-\Delta(t)]\{E_1+(E_2-E_1)\,\theta[x-\Delta(t)]\}$$

$$-\left\{\epsilon_1+(\epsilon_2-\epsilon_1)(E_2-E_1)\,\theta[x-\Delta(t)]\frac{\dot{\Delta}(t)}{c}\right\}, \tag{11}$$

$$(E_2-E_1)\,\delta[x-\Delta(t)]=(H_2-H_1)\frac{\dot{\Delta}(t)}{c}\,\delta[x-\Delta(t)].$$

Integration in the interval $x\in[\Delta(t)-0;\Delta(t)+0]$ then gives

$$(H_2-H_1)=(\epsilon_2-\epsilon_1)\frac{E_1+E_2}{2}\,\frac{\dot{\Delta}(t)}{c}+\frac{\epsilon_2+\epsilon_1}{2}(E_2-E_1)\frac{\dot{\Delta}(t)}{c},$$

$$(E_2-E_1)=(H_2-H_1)\frac{\dot{\Delta}(t)}{c}, \tag{12}$$

which can then be rewritten as

$$\frac{E_2}{E_1}=\frac{1-\left(\dfrac{\dot{\Delta}(t)}{c}\right)^2\epsilon_1}{1-\left(\dfrac{\dot{\Delta}(t)}{c}\right)^2\epsilon_2},$$

$$H_2-H_1=E_1\frac{(\epsilon_2-\epsilon_1)\dfrac{\dot{\Delta}(t)}{c}}{1-\left(\dfrac{\dot{\Delta}(t)}{c}\right)^2\epsilon_2}. \tag{13}$$

We thus arrive at the conclusion that the solution of the time-dependent master equation for a translated photonic crystal is equivalent to solving the stationary photonic crystal master Eq. (5) with the time dependent boundary conditions (13).

IV. COMPUTATIONAL METHOD AND APPROXIMATIONS

In practical applications, the characteristic velocity of a translated crystal is considerably smaller than the speed of light. Thus, $\dot{\Delta}(t)/c$ is a small parameter in our system. We can also reason that if one is interested in inducing transitions from one band of a photonic crystal to another by mechanical vibration, the driving frequency Ω inducing such a transition should be comparable to the characteristic band frequency ω, thus, $\dot{\Delta}(t)/c\sim\Delta\Omega/\lambda\omega\sim\Delta/R$, where R is a spatial period of the crystal that is of the order of the characteristic wavelength of an extended mode. This places us in a regime where the amplitude of vibrations is necessarily considerably smaller than a spatial period of a crystal and thus perturbation theory is clearly applicable. As the velocity of vibration is much smaller than the velocity of a propagating mode it is intuitive to expect that an instantaneous state of the system can be thought of as being composed of a superposition of modes for a stationary crystal but shifted by an amount equal to the current displacement $\Delta(t)$. We can put this assumption on a rigorous basis by employing a modified Bloch theorem appropriate for a rigidly translated photonic crystal. We prove (see the Appendix) that for the case of a rigidly translated photonic crystal, a time dependent solution of the electromagnetic fields still possesses a Bloch symmetry

$$\begin{pmatrix} H_q(x+R,t)\\ E_q(x+R,t)\end{pmatrix}=\exp(iqR)\begin{pmatrix} H_q(x,t)\\ E_q(x,t)\end{pmatrix}. \tag{14}$$

Thus, we can expand the magnetic field $H(x,t)$ in terms of the shifted eigenmodes $H_{q,\omega_n}[x-\Delta(t)]$ of the stationary master equation and obtain

$$H(x,t)_q=\sum_{\omega_n(q)}C_{q,\omega_n}(t)H_{q,\omega_n}[x-\Delta(t)]. \tag{15}$$

Here, the $C_{q,\omega_n}(t)$ are the time-dependent band population amplitudes (to be determined) and the $H_{q,\omega_n}(x)$ satisfy

$$\nabla\times\left[\frac{1}{\epsilon(x)}\nabla\times H_{q,\omega_n}(x)\right]=\frac{\omega_n^2(q)}{c^2}H_{q,\omega_n}(x). \tag{16}$$

We note that $H(x,t)_q$ satisfies the Bloch form of a solution and that the choice of the initial values of $C_{q,\omega_n}(0)$ and $\dot{C}_{q,\omega_n}(0)$ is made based on the boundary conditions. Thus, with a proper choice of $C_{q,\omega_n}(t)$, $H(x,t)_q$ is an exact solution of the time-dependent problem for the rigid translations of a photonic crystal.

Let us now develop equations for the time dependent population amplitudes $C_{q,\omega_n}(t)$.

We note from Eq. (16) that

$$\nabla\times\left\{\frac{1}{\epsilon[x-\Delta(t)]}\nabla\times H_{q,\omega_n}[x-\Delta(t)]\right\}$$

$$=\frac{\omega_n^2(q)}{c^2}H_{q,\omega_n}[x-\Delta(t)]. \tag{17}$$

In addition, the orthogonality of the modes gives

$$\langle H_{q,\omega_n}[x-\Delta(t)]|H_{q',\omega_{n'}}[x-\Delta(t)]\rangle=\delta_{n,n'}\delta_{q,q'}. \tag{18}$$

Substitution of $H(x,t)_q$ into the stationary photonic crystal master Eq. (5) and use of Eqs. (15), (16), and (17) leads to

$$\sum_{q,\omega_n(q)} C_{q,\omega_n}(t)H_{q,\omega_n}[x-\Delta(t)]\frac{\omega_n^2(q)}{c^2}$$

$$=-\frac{1}{c^2}\left\{\sum_{q,\omega_n(q)}\ddot{C}_{q,\omega_n}(t)H_{q,\omega_n}[x-\Delta(t)]\right.$$

$$+2\sum_{q,\omega_n(q)}\dot{C}_{q,\omega_n}(t)\frac{\partial H_{q,\omega_n}[x-\Delta(t)]}{\partial t}$$

$$\left.+\sum_{q,\omega_n(q)}C_{q,\omega_n}(t)\frac{\partial^2 H_{q,\omega_n}[x-\Delta(t)]}{\partial t^2}\right\}. \tag{19}$$

Now, using the orthogonality of the $H_{q,\omega_n}[x-\Delta(t)]$ modes we can rewrite the above equation in the form

$$0=\ddot{C}_{q,\omega_n}+\omega_n^2(q)C_{q,\omega_n}+2\sum_{q',\omega_{n'}(q')}\dot{C}_{q',\omega_{n'}}$$

$$\times\left\langle H_{q,\omega_n}[x-\Delta(t)]\left|\frac{\partial H_{q',\omega_{n'}}[x-\Delta(t)]}{\partial t}\right.\right\rangle$$

$$+\sum_{q',\omega_{n'}(q')}C_{q',\omega_{n'}}\left\langle H_{q,\omega_n}[x-\Delta(t)]\right.$$

$$\times\left|\frac{\partial^2 H_{q',\omega_{n'}}[x-\Delta(t)]}{\partial^2 t}\right\rangle. \tag{20}$$

Since the $H_{q,n}(x)$ are solutions of the stationary Master equation for a crystal, they are the Bloch waves of a stationary Hamiltonian and therefore

$$H_{q,\omega_n}[x-\Delta(t)]=\exp\{iq[x-\Delta(t)]\}U_{q,n}[x-\Delta(t)], \tag{21}$$

where $U_{q,n}[x-\Delta(t)]$ is a periodic function with periodicity R.

Given this particular form of the $H_{q,\omega_n}[x-\Delta(t)]$, we can express the transition matrix elements $\langle H_{q,\omega_n}[x-\Delta(t)]|\{\partial H_{q',\omega_{n'}}[x-\Delta(t)]/\partial t\}\rangle$ and $\langle H_{q,\omega_n}[x-\Delta(t)]|\{\partial^2 H_{q',\omega_{n'}}[x-\Delta(t)]/\partial t^2\}\rangle$ in terms of integrals over the derivatives of $U_{q,n}[x-\Delta(t)]$. To first order in $\Delta(t)$ one can derive

$$\left\langle H_{q,\omega_n}[x-\Delta(t)]\left|\frac{\partial H_{q',\omega_{n'}}[x-\Delta(t)]}{\partial t}\right.\right\rangle$$

$$=-[iq\delta_{n,n'}+M_{n,n'}(q)]\dot{\Delta}(t)\delta_{q,q'} \tag{22}$$

and

$$\left\langle H_{q,\omega_n}[x-\Delta(t)]\left|\frac{\partial^2 H_{q',n'}[x-\Delta(t)]}{\partial^2 t}\right.\right\rangle$$

$$=-[iq\delta_{n,n'}+M_{n,n'}(q)]\ddot{\Delta}(t)\delta_{q,q'}, \tag{23}$$

where $M_{n,n'}(q)$ is an antihermitian transition matrix defined as

$$M_{n,n'}(q)=\left\langle U_{q,n}\left|\frac{\partial U_{q,n'}}{\partial x}\right.\right\rangle_v \tag{24}$$

and the integral is taken over the volume of a unit cell v. Thus, from Eqs. (22) and (23) intraband transitions are not allowed in this formalism and only interband transitions are possible.

Substitution of Eqs. (22) and (23) back into Eqs. (20) then gives

$$\ddot{\overline{C}}-2\dot{\Delta}(t)(iq+M)\dot{\overline{C}}+[D-\ddot{\Delta}(t)(iq+M)]\overline{C}=0, \tag{25}$$

where $\overline{C}=[C_{q,\omega_0}(t),C_{q,\omega_1}(t),\ldots]$ is a vector of mode populations, and D is a diagonal matrix with squared natural mode frequencies on the diagonal, i.e., $\mathrm{diag}(D)=[\omega_0^2(q),\omega_1^2(q),\ldots]$. Finally, we note that Eq. (25) is not time reversal invariant, which is consistent with the fact that Eq. (5) represents a driven system.

V. RESULTS FOR RIGID TRANSLATIONS WITH A HARMONIC DISPLACEMENT

Let us now consider the special case of rigid vibrations with a harmonic displacement

$$\Delta(t)=\Delta\sin(\Omega t). \tag{26}$$

In this case, Eq. (25) for the time dependent population of modes $C_{q,\omega_n}(t)$ can be easily analyzed. Rewriting $\Delta(t)$ as

$$\Delta(t)=\Delta\frac{\exp(i\Omega t)-\exp(-i\Omega t)}{2i} \tag{27}$$

we notice that Eq. (25) allows a solution of the form

$$\overline{C}=\sum_{l=-\infty}^{+\infty}\overline{A}_{\omega+l\Omega}\exp[i(\omega+l\Omega)t]+\sum_{l=-\infty}^{+\infty}\overline{A}_{-\omega+l\Omega}$$

$$\times\exp[i(-\omega+l\Omega)t]. \tag{28}$$

Here, the eigenvectors $\overline{A}_{\omega+l\Omega}$ and $\overline{A}_{-\omega+l\Omega}$ are to be determined by substituting Eq. (28) into Eq. (25) and solving a complicated matrix equation.[13] Since we are dealing with small Δ [and we know that $\omega=\omega_n(q)$ when $\Delta=0$] the spectrum of the excited modes is not going to change much. Thus the frequencies for the excited modes can all be approxi-

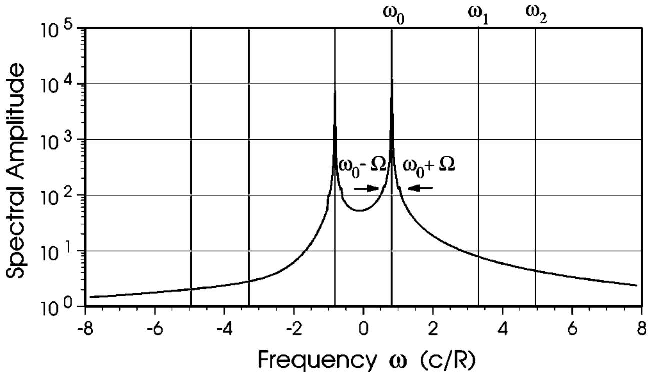

FIG. 3. Frequency spectrum of the population of band 0 at $q = \pi/2$ with $\Delta = 0.01$ and $\Omega = 0.2$. Note the dominating natural frequency harmonics with $\pm\omega_0(\pi/2)$ and excitations of the form $\pm\omega_0(\pi/2) + l\Omega$.

mated by $\omega_{excited} \simeq \pm\omega_n(q) + l\Omega$ where l is any integer and $\omega_n(q)$ are the normal frequencies of a stationary photonic crystal.

As an example, let us consider the case of a 1D photonic crystal with alternating dielectric slabs of width 0.8 and 0.2 and dielectric constants $\epsilon_1 = 1$ and $\epsilon_2 = 13$, respectively. For simplicity, we set $R = 1$ and $c = 1$ and concentrate on the mid zone wavevector $q = \pi/2$. The first three bands are easily calculated and have frequencies $\omega_0(\pi/2) = 0.8189$, $\omega_1(\pi/2) = 3.3047$, $\omega_2(\pi/2) = 4.9659$. For definiteness we focus on exploring a possible interband transition from band 0 to band 1, and thus set $\Omega_{res} = 2.4857$.

To investigate the time dependence of the excited states of the system we perform the following simulation. At $t = 0$ we initialize our state to be band 0 and calculate the time dependence of the band populations as we vibrate the crystal with a frequency Ω. We then analyze the Fourier spectra of the band populations.

Let us begin with the case $\Omega \ll \Omega_{res}$. A Fourier analysis of the time dependence of the populations for band 0 and band 1 gives the spectra shown in Figs. 3 and 4, respectively.

One can see in Fig. 3 that harmonics of the form $\pm\omega_0(\pi/2) + l\Omega$ are excited with $\pm\omega_0(\pi/2)$ having the dominant amplitude, which is in accordance with Eq. (28). Moreover, the excitations $\pm\omega_0(\pi/2) + l\Omega$ for $|l| > 1$ are so small they are unresolvable in the figures. Excitations $\pm\omega_0(\pi/2) \pm \Omega$ have Fourier components that are two orders of magnitude smaller than the Fourier components of the natural harmonics $\pm\omega_0(\pi/2)$. As the driving frequency is substantially smaller than the $0 \rightarrow 1$ resonant frequency, the transition to band 1 is suppressed and the amplitudes of excited modes in band 1 are at least three orders of magnitude smaller than the amplitudes of the $\pm\omega_0(\pi/2)$ natural modes in band 0. This is shown in Fig. 4. From the Fourier spectrum associated with band 1 one can see that the main harmonics are at $\pm\omega_1(\pi/2)$, $\pm\omega_1(\pi/2) \pm \Omega$, and $\pm\omega_0(\pi/2) \pm \Omega$. The presence of the latter frequencies reflects a ''memory'' of the band from which the transition originated.

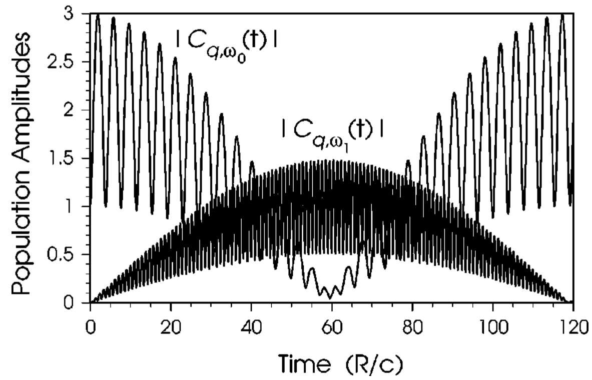

FIG. 5. Plot of the time-dependent population amplitudes for bands 0 and 1 at $q = \pi/2$ as a function of time t for the $0 \rightarrow 1$ interband transition with $\Delta = 0.01$, and $\Omega = \Omega_{res}$. With these parameters the population of band 1 becomes comparable with the population of band 0 at $t_{transition} \sim 60\frac{R}{c}$.

Let us now consider the case $\Omega = \Omega_{res}$. Tuning the frequency of vibration Ω to the interband frequency Ω_{res} leads to a strong interband coupling. After a characteristic transition time $t_{transition}$ the envelope of the amplitude of population of band 1 gradually approaches the same order of magnitude as the original population of band 0. This is shown in Fig. 5.

The rapid oscillations correspond to the natural frequencies of each band. Note that there is complete transfer at intervals of about $60\frac{R}{c}$. Fourier analysis of these spectra leads to the results shown respectively in Figs. 6 and 7. As in

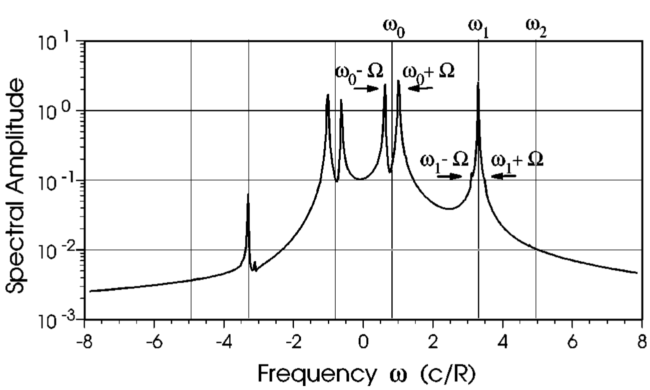

FIG. 4. Frequency spectrum of the population of band 1 at $q = \pi/2$ with $\Delta = 0.01$, and $\Omega = 0.2$. Natural frequency harmonics $\pm\omega_1(\pi/2)$ and their excitations $\pm\omega_1(\pi/2) + l\Omega$ are superimposed on the excitations $\pm\omega_0(\pi/2) + l\Omega$ induced by the $0 \rightarrow 1$ transition.

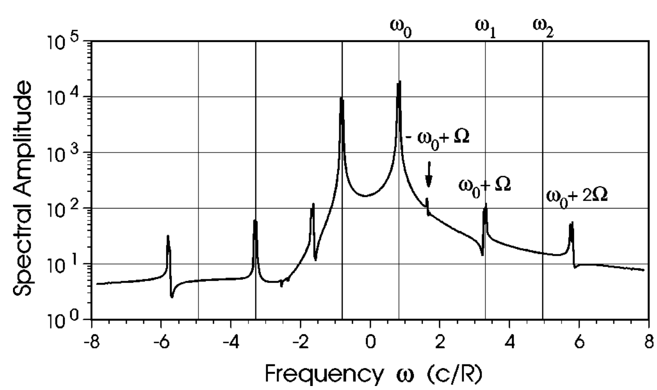

FIG. 6. Frequency spectrum of the population of band 0 at $q = \pi/2$ with $\Delta = 0.01$, $\Omega = \Omega_{res}$. As in the case of the off-resonance transitions the excitations in the band 0 are of the form $\pm\omega_0(\pi/2) + l\Omega$.

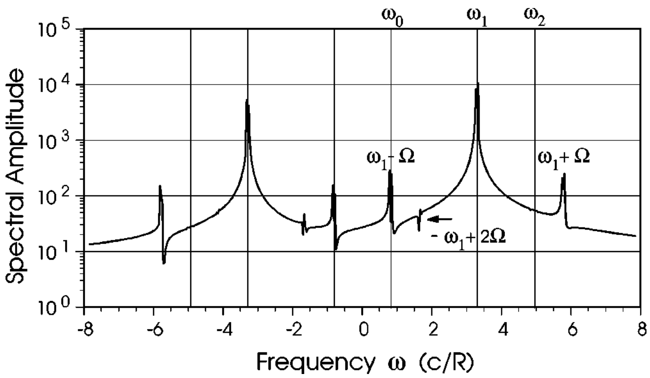

FIG. 7. Frequency spectrum of the population of band 1 at $q = \pi/2$ with $\Delta = 0.01$, $\Omega = \Omega_{res}$. Harmonics with the natural frequency of the band $\pm\omega_1(\pi/2)$ dominate the Fourier spectrum. Note that in this case the amplitude of $\pm\omega_1(\pi/2)$ is comparable to that of $\pm\omega_0(\pi/2)$ in Fig. 6.

the case of the off-resonance transitions, the general form of the excitations is $\pm\omega_{0,1}(\pi/2) + l\Omega$ where the amplitudes of the excitations with $|l| \geqslant 1$ are much smaller (at least one order of magnitude) than the amplitudes of the modes with the natural frequencies of the bands. Specifically, for band 0 (Fig. 6) it is clear that harmonics with the natural frequency of the band $\pm\omega_0(\pi/2)$ dominate the Fourier spectrum by two orders of magnitude. Similarly, for band 1 (Fig. 7) the harmonics $\pm[\omega_0(\pi/2) + \Omega_{res}]$ and $\pm\omega_1(\pi/2)$ coincide with each other and dominate the Fourier spectrum by at least one order of magnitude.

A natural issue to consider at this point concerns the interband transition time at resonance. This is addressed in detail in the next chapter.

VI. THE INTERBAND TRANSITION TIME

Since the amplitude of vibration plays the role of an interband coupling constant, one would expect that the transition time should be proportional to the inverse of this coupling constant. Doing standard time dependent perturbation theory on Eq. (25), with $t_{transition}$ defined to be the time required for the population amplitude of band 1 to reach its maximum, one obtains

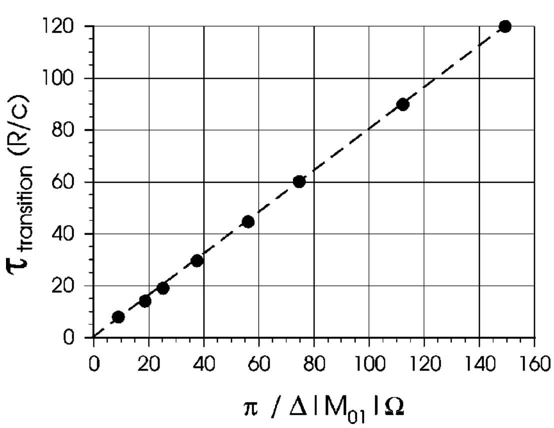

FIG. 8. Transition time versus $\pi/\Delta|M_{01}|\Omega$ for $q = \pi/2$ and $\Omega = \Omega_{res}$.

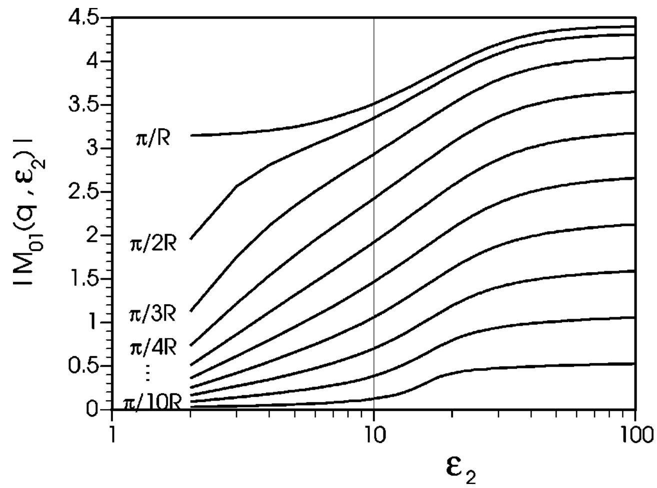

FIG. 9. Transition-matrix element for a set of Bloch wave vectors as a function of ϵ_2. Each curve corresponds to one of the Bloch wave vectors in a set $q = \pi/nR$ where $n = 1$ to 10.

$$t_{transition} = \frac{\sqrt{\omega_0\omega_1}}{\omega_0 + \omega_1}\,\frac{2\pi}{\Delta|M_{01}|\Omega_{res}}. \tag{29}$$

Here, $|M_{01}|$ is the absolute value of the transition-matrix element defined in Eq. (24). Since $\sqrt{\omega_0\omega_1}/(\omega_0 + \omega_1)$ will typically always be about 0.5 for a fairly wide range of ω_1/ω_0 we can approximate $t_{transition}$ for most practical purposes as

$$t_{transition} \sim \frac{\pi}{\Delta|M_{01}|\Omega_{res}}. \tag{30}$$

Note that this expression is also inversely proportional to the matrix element and resonant frequency. We shall return to examine this behavior shortly, but first we focus on the Δ-dependence. For $q = \pi/2$ and $\Omega = \Omega_{res}$ we determine $t_{transition}$ independently by varying the coupling constant and calculating the number of cycles needed for the amplitude of band 1 to reach its maximum. A comparison of these results with those predicted by Eq. (30) is given in Fig. 8.

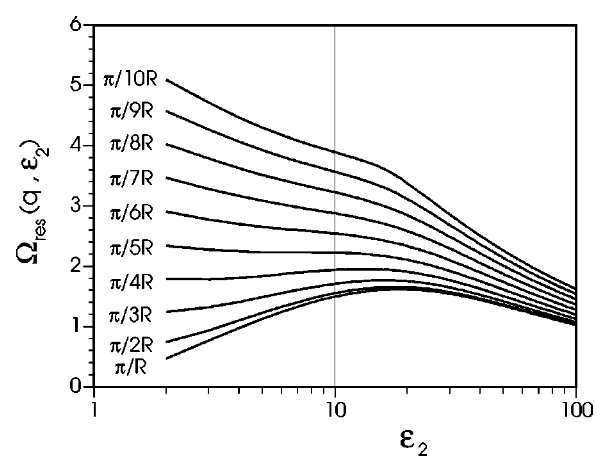

FIG. 10. Band $0 \to 1$ transition resonance frequency $\Omega_{res}(q)$ for a set of Bloch wave vectors as a function of ϵ_2. Each curve corresponds to one of the Bloch wave vectors in a set $q = \pi/nR$ where $n = 1$ to 10.

The results clearly show that $t_{transition}$ is inversely proportional to the amplitude of vibration and that Eq. (30) is a reasonable approximation.

We now turn to the case where Δ is fixed and $|M_{01}|$ and Ω_{res} are allowed to vary. Under these conditions the interband transition time is given by

$$t_{transition}(q,R_1,R_2,\epsilon_1,\epsilon_2) \sim \frac{\pi}{\Delta|M_{01}(q,R_1,R_2,\epsilon_1,\epsilon_2)|\Omega_{res}(q,R_1,R_2,\epsilon_1,\epsilon_2)}. \tag{31}$$

For simplicity, we begin by focusing on calculating $|M_{01}|$ and Ω_{res} in various limits. Earlier we considered a system with $\epsilon_1=1$, $\epsilon_2=13$, $R_1=0.8R$, and $R_2=0.2R$. Now we allow ϵ_2 to vary and calculate $|M_{01}(q,\epsilon_2)|$ for q in the interval $[0,\pi]$. The results are shown in Fig. 9.

One can see that for each value of q, the transition-matrix element is a monotonic function of ϵ_2 bounded from above by values independent of ϵ_2. One can show that these bounds are reached when $\epsilon_2 \gg \epsilon_1(R_1/R_2)^2$. In addition, it is relatively straightforward to show that in the limits of $\epsilon_2 \to \epsilon_1$ and $R_1 > R_2$, $|M_{01}(q,\epsilon_2)|$ becomes a function of the crystal structure and q alone. For example, at the band edge $q=\pi$, one can perform an analytical calculation of $|M_{01}(q)|$ and obtain

$$\lim_{\epsilon_2 \to \epsilon_1} M_{01}(\pi,\epsilon_2) = i\frac{\pi}{R} \tag{32}$$

and

$$\lim_{\epsilon_2 \gg \epsilon_1(R_1/R_2)^2} M_{01}(\pi,\epsilon_2) = i\frac{4\sqrt{2}\pi}{\sqrt{R_1}\left(\pi^2 - 4\frac{R_2}{R_1}\right)} \frac{\dfrac{\sin\left(2\sqrt{\frac{R_2}{R_1}}\right)}{1-\cos\left(2\sqrt{\frac{R_2}{R_1}}\right)}}{\sqrt{R_1 + \left[R_2 - \frac{\sqrt{R_1R_2}}{2}\sin\left(2\sqrt{\frac{R_2}{R_1}}\right)\right] \Big/ \left[1-\cos\left(2\sqrt{\frac{R_2}{R_1}}\right)\right]}}. \tag{33}$$

Thus, the extrema of $|M_{01}(\pi)|$ at a band edge can be tuned purely by geometric considerations. Finally, as it is clear from the plot, for the same value of ϵ_2, the maximum coupling between resonance bands is achieved at the band edge $q=\pi$ as one might expect to occur at a Bragg plane.

As far as the resonance frequency is concerned, it is plotted for each value of q in Fig. 10 as a function of ϵ_2. Note that for values of $q \gtrsim \pi/4R$, the $\Omega_{res}(q,\epsilon_2)$ exhibit an extremum at $\epsilon_2 \sim 18$, which is close to the quarter-wavelength condition $\epsilon_2 = \epsilon_1(R_1/R_2)^2 = 16$. At $\epsilon_2 \gg \epsilon_1(R_1/R_2)^2$ it is easy to show that interband resonance frequencies at all q tend to collapse, approaching 0 according to $\sim 1/\sqrt{\epsilon_2}$.

Using the results of Figs. 9 and 10, $t_{transition}$ can now be calculated for each value of q as a function of ϵ_2, as shown in Fig. 11. From the figure we see that for each value of q there exists an optimal value of ϵ_2 such that interband transition time is minimal. Another important fact is that for a fixed value of ϵ_2, the transition time will achieve its minimal value at the band edge $q=\pi$. The latter is partially a consequence of the fact that the coupling between states is maximal at the band edge. Thus, one would expect similar behavior at Bragg planes in general for other photonic crystal systems.

Finally, as a concrete example, let us consider transitions in the microwave regime. Here, the frequencies can be of the order of $\Omega \sim 10^9$ Hz. For the case of the photonic crystal considered in this paper with $\epsilon_1=1$, $\epsilon_2=13$, $R_1=0.8R$ and $R_2=0.2R$ the optimal transition time is given by

$$t_{transition} \sim 0.503\frac{R^2}{c\Delta}. \tag{34}$$

In the microwave regime $R \sim 0.1$ m and a reasonable amplitude for a displacement is $\Delta \sim 10^{-8}$ m, thus leading to an interband transition time of the order of $10^{-3} - 10^{-2}$ s.

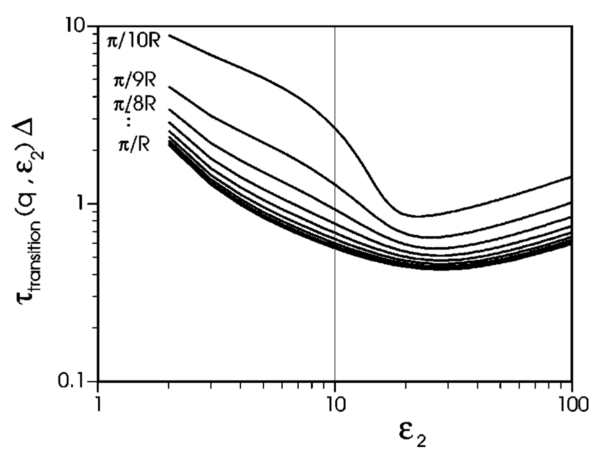

FIG. 11. Band $0 \to 1$ transition time for a set of Bloch wave vectors as a function of ϵ_2. Each curve corresponds to one of the Bloch wave vectors in a set $q=\pi/nR$ where $n=1$ to 10.

VII. SUMMARY

In this paper, we studied the behavior of electromagnetic states associated with photonic crystals, which are undergoing rigid time-dependent translations in position space. It was shown that the Bloch wave vector remains a conserved quantity and that an analogue of Bloch's theorem for a time dependent solution of the states can be formulated. It was also shown that under translations involving harmonic rigid vibrations of the photonic crystal, tuning the driving frequency to the interband resonance frequency, induces resonant transitions between the bands. In particular, a characteristic resonance transition time was derived, which scales inversely with the amplitude of vibrations, transition-matrix element and resonance frequency. Finally, it was established that given all the other parameters fixed an interband transition time is minimized if the transition is made at a Bragg plane.

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APPENDIX

In the following, we provide a detailed derivation of a modified Bloch theorem for a photonic crystal undergoing rigid time-dependent oscillations in position space. Consider the case of a photonic crystal vibrating with an amplitude $\Delta(t)=\Delta\sin(\Omega t)$. Maxwell's equations take the form

$$\nabla\times H(\overline{r},t)=\epsilon[\overline{r}-\overline{\Delta}(t)]\frac{\partial E[\overline{r},t)]}{\partial ct}+\frac{\partial\epsilon[\overline{r}-\overline{\Delta}(t)]}{\partial ct}E(\overline{r},t), \tag{A1}$$

$$\nabla\times E(\overline{r},t)=-\frac{\partial H(\overline{r},t)}{\partial ct},$$

where $\epsilon(\overline{r},t)$ is now a spatially periodic time dependent function such that $\exists\overline{R}\forall\,\overline{r}:\epsilon[\overline{r}+\overline{R}-\overline{\Delta}(t)]=\epsilon[\overline{r}-\overline{\Delta}(t)]$. It is reasonable to conjecture that this translational symmetry should still impose some restrictions on the fields even in the time-dependent case. In fact we will prove that Bloch's theorem still holds in the time-dependent case in the following form. For a time dependent solution of Eq. (A1) it is possible to define "good quantum numbers" $\overline{q}$ and ω_n so that

$$\begin{pmatrix} H_{\overline{q},\omega_n,\Omega}(\overline{r}+\overline{R},t) \\ E_{\overline{q},\omega_n,\Omega}(\overline{r}+\overline{R},t)\end{pmatrix}=\exp(i\overline{qR})\begin{pmatrix} H_{\overline{q},\omega_n,\Omega}(\overline{r},t) \\ E_{\overline{q},\omega_n,\Omega}(\overline{r},t)\end{pmatrix}. \tag{A2}$$

To demonstrate this, we look for a solution to a time dependent problem in a complete plain wave basis

$$\begin{pmatrix} H(\overline{r},t) \\ E(\overline{r},t)\end{pmatrix}=\int d\overline{q}d\omega\begin{pmatrix} H(\overline{q},\omega) \\ E(\overline{q},\omega)\end{pmatrix}|\overline{q},\omega\rangle, \tag{A3}$$

where

$$|\overline{q},\omega\rangle\equiv\frac{1}{(2\pi)^2}\exp(i\overline{qr}-i\omega t) \tag{A4}$$

and

$$\langle\overline{q}_0,\omega_0|\overline{q},\omega\rangle=\delta(\overline{q}_0-\overline{q})\,\delta(\omega_0-\omega). \tag{A5}$$

Since $\epsilon(\overline{r}+\overline{R})=\epsilon(\overline{r})$ we can decompose the dielectric function in terms of the reciprocal space modes

$$\epsilon(\overline{r})=\sum_G\epsilon_{\overline{G}}\exp(i\overline{Gr}). \tag{A6}$$

Substitution of Eqs. (A3) and (A6) into Maxwell's Eqs. (A1) gives

$$\int d\overline{q}d\omega i[H(\overline{q},\omega)\times\overline{q}]|\overline{q},\omega\rangle=\int d\overline{q}d\omega\frac{\partial\epsilon[\overline{r}-\overline{\Delta}(t)]}{\partial ct}E(\overline{q},\omega)|\overline{q},\omega\rangle-\int d\overline{q}d\omega i\epsilon[\overline{r}-\overline{\Delta}(t)]\frac{\omega}{c}E(\overline{q},\omega)|\overline{q},\omega\rangle, \tag{A7}$$

$$\int d\overline{q}d\omega i[E(\overline{q},\omega)\times\overline{q}]|\overline{q},\omega\rangle=-\int d\overline{q}d\omega iH(\overline{q},\omega)\frac{\omega}{c}|\overline{q},\omega\rangle.$$

Multiplying both sides of Maxwell's equations by $\langle\overline{q}_0,\omega_0|$ we get

$$0=H(\overline{q}_0,\omega_0)\times\overline{q}_0+\int d\overline{q}d\omega\frac{\omega}{c}E(\overline{q},\omega)\langle\overline{q}_0,\omega_0|\epsilon[\overline{r}-\overline{\Delta}(t)]|\overline{q},\omega\rangle+i\int d\overline{q}d\omega E(\overline{q},\omega)\langle\overline{q}_0,\omega_0|\frac{\partial\epsilon[\overline{r}-\overline{\Delta}(t)]}{\partial ct}|\overline{q},\omega\rangle, \tag{A8}$$

$$0=E(\overline{q}_0,\omega_0)\times\overline{q}_0+\frac{\omega_0}{c}H(\overline{q}_0,\omega_0).$$

1. Evaluation of $\int d\bar{q}d\omega\omega/cE(\bar{q},\omega)\langle\bar{q}_0,\omega_0|\epsilon[\bar{r}-\bar{\Delta}(t)]|\bar{q},\omega\rangle$

First, let us investigate the form of the term

$$\int d\bar{q}d\omega\frac{\omega}{c}E(\bar{q},\omega)\langle\bar{q}_0,\omega_0|\epsilon[\bar{r}-\bar{\Delta}(t)]|\bar{q},\omega\rangle \tag{A9}$$

more closely. Since

$$\epsilon[\bar{r}-\bar{\Delta}(t)]=\sum_{\bar{G}}\epsilon_{\bar{G}}\exp[i\overline{Gr}-i\overline{G\Delta}(t)] \tag{A10}$$

we have

$$\int d\bar{q}d\omega\frac{\omega}{c}E(\bar{q},\omega)\langle\bar{q}_0,\omega_0|\epsilon[\bar{r}-\bar{\Delta}(t)]|\bar{q},\omega\rangle \tag{A11}$$

$$=\int d\bar{q}d\omega\frac{\omega}{c}E(\bar{q},\omega)\sum_{\bar{G}}\epsilon_{\bar{G}}\langle\bar{q}_0,\omega_0|\times\exp[i\overline{Gr}-i\overline{G\Delta}(t)]|\bar{q},\omega\rangle. \tag{A12}$$

But now

$$\langle\bar{q}_0,\omega_0|\exp[i\overline{Gr}-i\overline{G\Delta}(t)]|\bar{q},\omega\rangle=\frac{1}{(2\pi)^4}\int d\bar{r}\exp[i(\bar{q}+\bar{G}-\bar{q}_0)\bar{r}]\times\int dt\exp[i(\omega_0-\omega)t-i\overline{G\Delta}(t)] \tag{A13}$$

and the integral over space is trivial. Thus, we obtain

$$\langle\bar{q}_0,\omega_0|\exp[i\overline{Gr}-i\overline{G\Delta}(t)]|\bar{q},\omega\rangle=\delta(\bar{q}+\bar{G}-\bar{q}_0)\frac{1}{2\pi}\int dt\exp[i(\omega_0-\omega)t-i\overline{G\Delta}(t)]. \tag{A14}$$

The time-dependent integral can be performed analytically in the following way:

$$\frac{1}{2\pi}\int dt\exp[i(\omega_0-\omega)t-i\overline{G\Delta}(t)]=\frac{1}{2\pi}\int dt\exp[i(\omega_0-\omega)t]\left\{\sum_{l=0}^{+\infty}\frac{(-i)^l}{l!}[\overline{G\Delta}(t)]^l\right\}. \tag{A15}$$

Now assume that the crystal is shaken with a single frequency

$$\bar{\Delta}(t)=\bar{\Delta}\frac{\exp(i\Omega t)-\exp(-i\Omega t)}{2i}. \tag{A16}$$

Substitution of Eq. (A16) into Eq. (A15) leads to the following

$$\frac{1}{2\pi}\int dt\exp[i(\omega_0-\omega)t-i\overline{G\Delta}(t)] \tag{A17}$$

$$=\frac{1}{2\pi}\int dt\exp[i(\omega_0-\omega)t]\left\{\sum_{n=0}^{+\infty}\frac{(-1)^l}{l!}\left(\frac{\overline{G\Delta}}{2}\right)^l\sum_{p=0}^{l}\times(-1)^p\exp[i\Omega t(l-p)]\exp(-i\Omega tp)\frac{l!}{p!(l-p)!}\right\}. \tag{A18}$$

The integration over time can now be easily performed to give

$$\frac{1}{2\pi}\int dt\exp[i(\omega_0-\omega)t-i\overline{G\Delta}(t)]=\sum_{l=-\infty}^{+\infty}\delta(\omega_0-\omega+l\Omega)D_l(\overline{G\Delta}), \tag{A19}$$

where coefficients $D_l(\overline{G\Delta})$ can be easily calculated by collecting the powers of $\exp(i\Omega t)$ to obtain

$$D_{2l}=D_{-2l}=(-1)^l\sum_{f=l}^{+\infty}\frac{(-1)^f}{2^{2f}}\frac{(\overline{G\Delta})^{2f}}{(f-l)!(f+l)!} \tag{A20}$$

and

$$D_{2l+1}=-D_{-2l-1}=(-1)^{l+1}\sum_{f=l}^{+\infty}\frac{(-1)^f}{2^{2f+1}}\frac{(\overline{G\Delta})^{2f+1}}{(f-l)!(f+l+1)!}. \tag{A21}$$

Finally, we arrive at the following expression:

$$\int d\bar{q}d\omega\frac{\omega}{c}E(\bar{q},\omega)\sum_{\bar{G}}\epsilon_{\bar{G}}\langle\bar{q}_0,\omega_0|\exp[i\overline{Gr}-i\overline{G\Delta}(t)]|\bar{q},\omega\rangle \tag{A22}$$

$$=\frac{1}{c}\sum_{\bar{G}}\epsilon_{\bar{G}}\left[\sum_{l=-\infty}^{+\infty}(\omega_0+l\Omega)\times E(\bar{q}_0-\bar{G},\omega_0+l\Omega)D_l(\overline{G\Delta})\right]. \tag{A23}$$

2. Evaluation of $i\int d\bar{q}d\omega E(\bar{q},\omega)\langle\bar{q}_0,\omega_0|\{\partial\epsilon[\bar{r}-\bar{\Delta}(t)]/\partial ct\}|\bar{q},\omega\rangle$

Proceeding in exactly the same fashion as in the previous section, one can easily derive the following expression:

$$i\int d\bar{q}d\omega E(\bar{q},\omega)\left\langle\bar{q}_0,\omega_0\left|\frac{\partial\epsilon[\bar{r}-\bar{\Delta}(t)]}{\partial ct}\right|\bar{q},\omega\right\rangle \tag{A24}$$

$$=\frac{1}{c}\sum_{\bar{G}}\frac{(\overline{G\Delta})}{2}\Omega\epsilon_{\bar{G}}\left\{\sum_{l=-\infty}^{+\infty}E(\bar{q}_0-\bar{G},\omega_0+l\Omega)\times[D_{l+1}(\overline{G\Delta})-D_{l-1}(\overline{G\Delta})]\right\}. \tag{A25}$$

3. Solution of the time-dependent Maxwell's equation for the rigid harmonic vibrations of a photonic crystal

Combining the results of the above sections we arrive at the following form of a solution for Maxwell's equations:

$$0=H(\overline{q_0},\omega_0)\times\overline{q_0}+\frac{1}{c}\sum_{\overline{G}}\epsilon_{\overline{G}}\left\{\sum_{l=-\infty}^{+\infty}E(\overline{q_0}-\overline{G},\omega_0+l\Omega)\left[\frac{(\overline{G\Delta})}{2}\Omega(D_{l-1}(\overline{G\Delta})-D_{l+1}(\overline{G\Delta}))+(\omega_0+l\Omega)D_l(\overline{G\Delta})\right]\right\},$$

$$0=E(\overline{q_0},\omega_0)\times\overline{q_0}+\frac{\omega_0}{c}H(\overline{q_0},\omega_0). \tag{A26}$$

A striking conclusion that can be drawn from this form of a solution is that it is still possible to define a ''good'' quantum number $\overline{q_0}$ for a vibrating photonic crystal, since the modes with different $\overline{q_0}$ do not mix. Another immediate conclusion is that along with a normal mode frequency ω_0 and amplitude

$$\begin{pmatrix} H(\overline{q_0},\omega_0) \\ E(\overline{q_0},\omega_0) \end{pmatrix}$$

the harmonics of the ''driven'' modes will be also excited with frequencies $\omega_0+l\Omega$ and amplitudes

$$\begin{pmatrix} H(\overline{q_0},\omega_0+l\Omega) \\ E(\overline{q_0},\omega_0+l\Omega) \end{pmatrix}.$$

Thus a solution to a time-dependent Hamiltonian equation can be written in the form

$$\begin{pmatrix} H_{\overline{q_0},\omega_0}(\overline{r},t) \\ E_{\overline{q},\omega_0}(\overline{r},t) \end{pmatrix}=\sum_{\overline{G}}\sum_{l=-\infty}^{+\infty}\begin{pmatrix} H(\overline{q_0}-\overline{G},\omega_0+l\Omega) \\ E(\overline{q_0}-\overline{G},\omega_0+l\Omega) \end{pmatrix}\times\exp[i(\overline{q_0}-\overline{G})\overline{r}-i(\omega_0+l\Omega)]. \tag{A27}$$

After a set of simple manipulations the final form of the solution is

$$\begin{pmatrix} H_{\overline{q_0},\omega_0}(\overline{r},t) \\ E_{\overline{q_0},\omega_0}(\overline{r},t) \end{pmatrix}=\exp(i\overline{q_0}\overline{r})\overline{U}_{\overline{q_0},\omega_0,\Omega}(\overline{r},t),$$

$$\overline{U}_{\overline{q_0},\omega_0,\Omega}(\overline{r}+\overline{R},t)=\overline{U}_{\overline{q_0},\omega_0,\Omega}(\overline{r},t), \tag{A28}$$

where

$$\overline{U}_{\overline{q_0},\omega_0,\Omega}(\overline{r},t)=\sum_{l=-\infty}^{+\infty}\tilde{U}_{\overline{q_0},\omega_0,\Omega,l}(\overline{r})\exp[-i(\omega_0+l\Omega)t],$$

$$\tilde{U}_{\overline{q_0},\omega_0,\Omega,l}(\overline{r}+\overline{R})=\tilde{U}_{\overline{q_0},\omega_0,\Omega,l}(\overline{r}),$$

$$\tilde{U}_{\overline{q_0},\omega_0,\Omega,l}(\overline{r})=\sum_{\overline{G}}\begin{pmatrix} H(\overline{q_0}-\overline{G},\omega_0+l\Omega) \\ E(\overline{q_0}-\overline{G},\omega_0+l\Omega) \end{pmatrix}\exp(-i\overline{Gr}). \tag{A29}$$

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[13] Note that $|\overline{A}_{\omega+l\Omega}|\neq|\overline{A}_{-\omega-l\Omega}|$ in general since we are not dealing with a time-reversal invariant system.