Fabrication and THz loss measurements of porous subwavelength fibers using a directional coupler method

Alexandre Dupuis$^1$, Jean-François Allard$^2$, Denis Morris$^2$, Karen Stoeffler$^1$, Charles Dubois$^1$, and Maksim Skorobogatiy$^1$

www.photonics.phys.polymtl.ca

$^1$ École Polytechnique de Montréal, C.P. 6079, Centre-Ville Montreal, QC H3C 3A7, Canada

$^2$ University of Sherbrooke, Department of Physics, Sherbrooke, QC J1K 2R1, Canada

Abstract: We report several strategies for the fabrication of porous subwavelength fibers using low density Polyethylene plastic for low-loss terahertz light transmission applications. We also characterize transmission losses of the fabricated fibers in terahertz using a novel non-destructive directional coupler method. Within this method a second fiber is translated along the length of the test fiber to probe the power attenuation of a guided mode. The method is especially suitable for measuring transmission losses through short fiber segments, a situation in which standard cutback method is especially difficult to perform. We demonstrate experimentally that introduction of porosity into a subwavelength rod fiber, further reduces its transmission loss by as much as a factor of 10. The lowest fiber loss measured in this work is $0.01\text{cm}^{-1}$ and it is exhibited by the 40% porous subwavelength fiber of diameter 380 $\mu\text{m}$. For comparison, the loss of a rod-in-the-air subwavelength fiber of a similar diameter was measured to be $\sim 0.1\text{cm}^{-1}$, while the bulk loss of a PE plastic used in the fabrication of such fibers is $\gtrsim 1\text{cm}^{-1}$. Finally, we present theoretical studies of the optical properties of individual subwavelength fibers and a directional coupler. From these studies we conclude that coupler setup studied in this paper also acts as a low pass filter with a cutoff frequency around 0.3THz. Considering that the spectrum of a terahertz source used in this work falls off rapidly below 0.25THz, the reported loss measurements are, thus, the bolometer averages over the $\sim 0.25\text{THz} - 0.3\text{THz}$ region.

© 2009 Optical Society of America

OCIS codes: (040.2235) Far infrared or terahertz; (060.2280) Fiber design and fabrication; (060.1810) Buffers, couplers, routers, switches, and multiplexers

References and links

In the past several years there have been promising technological demonstrations exploiting the terahertz (THz) frequency range for bio-medical sensing, spectroscopy, and non-invasive imaging applications. However, designing efficient waveguides for remote delivery of broadband THz radiation remains a challenging problem due to high absorption losses of most materials in this spectral range. Many different designs have been investigated, including all-dielectric,\textsuperscript{1–3} and metallized waveguides\textsuperscript{4–6} guiding by total internal reflection,\textsuperscript{1–3, 7} metallic reflection,\textsuperscript{8–10} and photonic band gap.\textsuperscript{11–13} Of the many designs that have recently been explored, subwavelength dielectric\textsuperscript{1} and subwavelength metallic wires,\textsuperscript{4} are among those with the lowest reported propagation losses (e.g. $\alpha \simeq 0.01 \text{ cm}^{-1}$ at $\omega = 0.3 \text{ THz}$). The basic strategy for reducing the waveguide propagation loss is to guide predominantly in the air because of its lower material absorption loss. Thus, subwavelength dielectric wires act as a fiber core with a substantial fraction of transmitted power guided in the surrounding air cladding. Such wires have very delocalized modes, extending far into the air cladding, and are, thus, subjected to high cross-talk and large bending losses. The metallic wires guide using a Somerfeld wave confined at the wire/air interface. Such a wave is also strongly delocalized in the air cladding, and thus...
is subject to the similar limitations as does a subwavelength dielectric wire. Additional complication for the metallic wires is that the guided mode is an azimuthally polarized surface plasmon making it difficult to excite with a regular linearly polarized emitter radiation. Overall, the simplicity of the fabrication process, and ease of input coupling has made the dielectric subwavelength fiber an attractive technology for guiding THz light.

Recently, our group\textsuperscript{3, 14} has proposed an improvement over the basic subwavelength fiber design by making the fiber core porous. Similar suggestion has later appeared also in.\textsuperscript{15} The idea of adding holes into a subwavelength fiber is our extension of the earlier work of Nagel et al.\textsuperscript{16} In that work the authors have demonstrated that addition of a single subwavelength hole into the center of a solid fiber core can result in strong field confinement inside such a hole. In this paper we consider a particular porous fiber design\textsuperscript{3} in which the fiber core diameter is comparable to the wavelength of transmitted light, while a periodic array of subwavelength holes is inserted into the fiber core. Numerical simulations have demonstrated that the major portion of THz power launched into such a porous fiber is confined within the air holes inside of the fiber core. As a result, coupling to the cladding environment is greatly reduced, while the modal absorption loss can be further reduced by a factor of 10 over the loss of a standard subwavelength rod-in-the-air fiber of similar diameter. Moreover, when comparing porous fiber to the rod-in-the-air subwavelength fiber exhibiting the same transmission loss due to material absorption, we found that the effective refractive index of the porous fiber is significantly higher than that of the rod-in-the-air fiber. Thus, the fundamental mode of a porous fiber is confined stronger inside the fiber core, and it has a smaller modal area compared to the one of the fundamental mode of a comparable in loss standard subwavelength fiber. This, in turn, suggests that interaction with the cladding environment, as well as bending loss in porous fibers is much smaller than that in the rod-in-the-air fibers of comparable losses. Finally, as we demonstrate at the end of the paper, dispersion of a subwavelength porous fiber is much smaller than the dispersion of a standard rod-in-the-air fiber of similar diameter, which is again a major plus for this new type of subwavelength fibers.

In this paper, we report on the fabrication and transmission loss measurements of several porous subwavelength fibers, as well as rod-in-the-air fibers. We also present a novel and non-destructive directional coupler (DC) method to measure the fiber transmission loss, thus circumventing many technical problems encountered with the standard cutback measurement methodology. We then demonstrate experimentally that porous fibers generally exhibit much smaller transmission losses than those of the rod-in-the-air fibers of comparable diameters. Finally, we present finite element method (FEM) and semi-analytical simulations of the optical properties of the porous and rod-in-the-air subwavelength fibers, as well as a coupler based on such fibers. We further confirm our experimental findings with theoretical analysis.

2. Preform and fiber fabrication

All the fibers in this work were fabricated with linear low density polyethylene, Grade Sclair FP120A, purchased from Nova Chemicals. Polyethylene (PE) is one of the polymers with the lowest material absorption losses\textsuperscript{17} ($\alpha < 2.5 \text{ cm}^{-1}$ for $\omega < 2 \text{ THz}$). The test fiber diameters reported below are the averages of the fiber diameters at the two fiber ends. Variation of the test fiber diameter over its length was less than $\pm 20 \mu$m for most of the fibers. The length of all the test fibers was around 25cm.

Porous fibers were made using two different fabrication techniques. The first method is the standard tube stacking technique. To achieve a fiber with a thin network of veins it is preferable to stack thin-walled tubes when making the fiber preform. Since such tubes were not available commercially, we fabricated thin-walled tubes (straws) by rolling 100 $\mu$m thick PE film within a 5 mm inner diameter metal tube followed by solidification and annealing in the oven. The
resulting tubes had inner and outer diameters of 4.25 and 5 mm, respectively. Thus fabricated straws were subsequently stacked and an outer layer of PE film was rolled around the stack (see the preform picture referred to as "PE Tubes" in Fig. 1). The ends of the straws were sealed with epoxy glue to prevent hole collapse during drawing. This preform was then drawn into fiber at 145°C and drawing speed of 0.5 m/min. Whereas PE becomes soft at $T_g \sim 110^\circ$C, it only melts and becomes drawable at higher temperatures. Trapping of air within the tubes only prevents the complete hole collapse, while it does not prevent partial hole collapse (relative reduction of the hole size compared to the fiber diameter), which is an inherent feature of a drawing process, unless active hole pressurization is used. Therefore this type of fiber was drawn at a rather low
temperature (145°C) in an attempt to reduce hole collapse (partial collapse). Cross-sections of the resultant fibers, designated as PE Tubes, are presented in Figs. 1(a)-(c). Using this fabrication method the porous microstructure was clearly achieved, however, the hole collapse was significant resulting in thick material veins.

The second method for the fabrication of porous PE fibers is a subtraction technique where a part of a drawn all-solid fiber is dissolved in order to form air holes. We chose Poly(methyl methacrylate) (PMMA) as a sacrificial polymer as it can be easily dissolved in the tetrahydrofuran (THF) solvent while leaving the PE plastic intact. The preform was fabricated in the following manner. Seven PMMA rods (6.35 mm diameter), purchased from McMaster-CARR, were placed in a hexagonal array within a polytetrafluoroethylene (PTFE) tube (2.54 cm inner diameter) and the interstitial regions were filled with PE granules. Upon heating in the oven at 210°C, the PE granules densified around the PMMA tubes by forming a composite preform (see the preform picture referred to as “PE/PMMA” in Fig. 1). This preform was subsequently drawn into fiber at 210°C and drawing speed of 0.8 m/min. Note that this arbitrarily high temperature (210°C) was much higher than necessary (for PMMA $T_g \sim 140^\circ C$) in order to insure that both polymers were melted enough to draw readily. Fiber segments of different lengths and diameters were then submerged into the THF solvent for several days to etch away the PMMA inclusions. The fiber segments were subsequently left to dry for several days to allow solvent extraction by evaporation from within the fibers. Resulting fibers showed clear porous microstructure with thin material veins. Cross-sections of these fibers, designated as "PE/PMMA", are presented in Figs. 1(d)-(f). The advantage of this technique is that hole collapse during drawing is avoided as fiber preform contains no holes.

A non porous fiber was fabricated using the same PE material as in the case of the two other fibers. Granules were melted in a PTFE tube and compacted to remove air (see the preform picture referred to as "Non Porous” in Fig. 1). The preform was subsequently drawn into fiber at 160°C and drawing speed of 0.6 m/min. The rod-in-the-air fibers (designated as "Non Porous" in Figs. 1(g)-(i)) were then used as a benchmark for comparison with the porous fibers.

3. Directional coupler method for measuring fiber transmission loss

A standard technique for measuring fiber transmission loss is a cutback method. Within this method a direct power transmission through the fibers of different lengths is measured. The input end of the test fiber is fixed during the procedure, and all the cuts are performed at the fiber output end. By plotting transmission intensity versus fiber length on a log-log plot one can extract the fiber loss coefficient. The main advantage of this procedure is that it eliminates the need to estimate the input coupling efficiency into the waveguide. While being a very powerful method for measuring losses in telecommunication fibers, the standard cutback method have to be modified to be applicable in the case of short fibers exhibiting high losses.

First of all, we note that the reference (background) signal, measured in the absence of a fiber, must be subtracted from the total signal, measured with the fiber in place, in order to obtain only the fiber contribution to the transmitted signal. In a typical cut-back setup where light source and a detector are well separated, such a background signal typically falls into the noise floor of a detector. However, in case of short fiber segments placed along a straight line between the light source and detector, the background signal due to direct overlap of a remaining beam from the light source with a detector can be significant. If the background signal cannot be ignored, a direct approach is to attempt blocking the part of a beam not coupled into a waveguide by using an aperture, or by inclining the waveguide so that the detector is out of the line of sight of a light source. If satisfactory blocking of the stray light from a light source is problematic one has to measure the background directly and subtract it from the measurement. In the case of short length (~25cm) subwavelength fibers that we sought to measure, the level of stray
light reaching the detector was relatively high, and our attempts to screen it were unsuccessful because of the interference of a screen with highly delocalized in the air modes of the fibers. Therefore, we have resorted to direct measurements of the background signal.

Secondly, the cutback technique is a destructive method and is prone to errors due to variations in the cleave quality and realignment of the fiber output end. This problem is especially pronounced when working with a large input aperture THz bolometer. Because of the bolometer internal optics, the amount of light actually sent onto the sensor surface is strongly dependent
on the distance and angle of the fiber output end with respect to the bolometer input aperture.
Despite significant efforts at trying to develop a systematic procedure for repositioning the fiber output end at the input aperture of a bolometer to obtain reproducible measurements we found this approach impractical.

To circumvent the two above mentioned difficulties we have devised a novel loss measurement method, which is both reliable and non-destructive. The method consists of using the second subwavelength fiber (further referred to as a coupler fiber) to form a directional coupler with a test fiber. The output end of a coupler fiber is fixed with respect to the bolometer input aperture. Such a directional coupler (coupler fiber and bolometer) is then translated along the length of a test fiber in order to probe the test fiber power attenuation (see Fig. 2). When translating directional coupler along the test fiber, power coupled into a coupler fiber will decrease as the length of a test fiber increases (because of the test fiber propagation loss). This method is similar in spirit to the evanescent prism coupling technique that had been used by Boudrioua et al. to measure transmission loss of planar waveguides. We would like to mention that a directional coupler featuring two touching subwavelength fibers has recently been used in a THz imaging setup, however coupler geometry was fixed during those measurements.

Fig. 2(a) presents schematics of the directional coupler setup. A Ti:sapphire laser beam is focused onto a photoconductive antenna to emit THz light. The THz emission spectrum was relatively broad (inset in Fig. 2(b)), extending between 0.25THz and 2.15THz (10% of the maximum power) with an emission maximum at 0.75THz. The THz radiation is collected and refocused by the parabolic mirrors in order to couple light into a test fiber. The two ends of a test fiber are positioned individually with 3-axis mounts. Moreover, the fiber can be accurately aligned with the optical axis of a THz beam with the help of a Ti:sapphire laser beam, which passes through a small hole in the PM2 parabolic mirror. The coupler fiber and a bolometer are both mounted on a rail, which can be translated along the length of a test fiber with an accuracy of 0.5mm. The coupler fiber is a 29 cm long non-porous PE fiber with an average diameter of 380μm. The directional coupler consists of a straight 7cm-long segment of a coupler fiber which was brought in parallel to and almost touching the test fiber. Outside of the coupler region, the coupler fiber is angled off by 16° towards the bolometer (see Fig. 2). The test fiber was placed at an angle of 10° with respect to the optical axis. The two angles were chosen as to insure that the bolometer remained outside of the divergence cone of THz light, while there was still enough light passing through the bend in the coupler fiber. As was confirmed by the calibration experiments only the light guided by the coupler fiber could reach the detector, since the bolometer was registering the normal noise level when coupling fiber was removed. This approach has eliminated all the reference (background) signal problems encountered with the standard cutback technique. As an example, when both the coupler fiber and the test fiber were removed, the bolometer measured 6μV of the noise level. When only the coupler fiber was used, the bolometer measured ~100μV as the reference signal. Finally, when both the coupler and the test fibers were used, the bolometer measured ~800μV, which results in a signal to noise ratio of over 20dB.

During the course of the experiment we have discovered that it is critical for the subwavelength fibers to be held straight in order to avoid strong bending losses. Design of fiber holders in the case of subwavelength fibers is challenging as such holders must insure low interference with highly delocalized modal fields, while still holding the fibers strongly enough so that enough tension could be applied to keep the fibers straight. Some earlier papers have proposed using small holes in a paper sheet or a PE film to pinch an inserted fiber in place. However, it is difficult to find just the right hole size to be able to insert the fiber into the hole, while also holding the fiber tightly. Moreover, fibers of different diameters require different holders. After experimenting with different fiber holders we believe a better solution is to tie a knot on
the fiber using a thin sewing thread. A knot holds the fiber tightly, remains low loss because of the small diameter of the thread, and easily accommodates fibers of different diameter. Note, however, that asymmetrical knots can apply torsion and bend the fiber tip. We therefore used a symmetrical Fisherman’s Knot and inserted the fiber between its two constituent simple knots (see inset in Fig. 2(c)). This technique was used to hold all the fibers. Finally, two knots were used as separators between the test and coupler fibers to make sure that the distance between fibers in the coupler remained the same even when the coupler fiber was displaced along the test fiber. The setup was aligned to ensure that during translations of the coupler the test fiber was always just-touching the knots on the coupler fiber, while no extra bending was introduced at the coupler input and output ends (see Fig. 2 (c)). Finally, the knot size and, thus, the air gap between the two fibers was estimated to be $\sim 400 \mu m$ in all the experiments.

Fig. 3 presents an example of the power attenuation measurement performed using the directional coupler method. Fig. 3.(a) shows variation of power as a function of displacement ($\Delta z$) along the length of a test fiber. Total and reference signals are measured with and without the test fiber, respectively. b) Fit of the fiber attenuation data ($P_{\text{Fiber}} = P_{\text{Total}} - P_{\text{Ref}}$).

To understand how to interpret the experimental data presented in Fig. 3, let us begin by considering the case of a fixed wavelength at which both the test fiber and the coupler fiber...
are single moded. We denote $P_{CF}(z)$ to be the power measured at the end of a coupler fiber translated over the distance $z$ along the test fiber. We denote $P_{TF}^0$ to be the total power coupled into the test fiber. We further assume that the coupling efficiency $\eta$ between the two fibers is constant along the whole length of a test fiber. Under these assumptions the test fiber attenuation coefficient $\alpha_{TF}$, in principle, can be calculated from only two measurements:

$$
\begin{align*}
P_{CF}(z_1) &= \eta \cdot P_{TF}^0 \cdot \exp(-\alpha_{TF} \cdot (z_1 - \alpha_{CF} \cdot z_{CF})) \\
\ln \left( \frac{P_{CF}(z_2)}{P_{CF}(z_1)} \right) &= \frac{1}{z_2 - z_1} \ln \left( \frac{P_{CF}(z_1)}{P_{CF}(z_2)} \right) \nonumber
\end{align*}
$$

where $\alpha_{CF}$ is the loss of a coupler fiber and $z_{CF}$ is the length of a coupler fiber. Equation (1) is a simple scaling law that describes a line on a semilog scale. While the strict minimum for fitting a line is two points, such a fit will be more accurate on multiple experimental points. The main advantage of the directional coupler method is that it is straightforward to measure many data points by simply displacing the coupler fiber along the test fiber (no cleaving or realignment is required). This is an important part of the validation process as one can easily measure as many points as deemed necessary for a satisfactory fit. Secondly, it should be noted that the use of a linear least squares fit yields maximum likelihood limits for the fitted slopes, which give a more accurate evaluation of the attenuation constant error bounds. These facts lead us to believe that the directional coupler method increases the accuracy and reliability of the loss estimates in comparison to the propagation losses of subwavelength fibers previously determined by the cutback method.\(^1\)

Strictly speaking, the data interpretation method expressed by (1) is only valid for a frequency resolved measurement. In our case we use a bolometer as a detector, therefore we could only register the spectral average of the power attenuation. This consideration is modified by the fact that the 29cm-long coupler fiber (rod-in-the-air fiber), as well as the coupler itself acted effectively as a low pass filter. Indeed, our theoretical simulations presented later in the paper show that two subwavelength fibers of 380$\mu$m diameter separated by 380$\mu$m air gap couple strongly (at least 50% by power) only at frequencies smaller than $\sim 0.3$THz, due to the otherwise strong confinement of the modal fields within the fiber cores. Moreover, absorption loss of the rod-in-the-air coupler fiber becomes sufficiently low ($\lesssim 0.1$cm$^{-1}$) only at frequencies below $\sim 0.3$THz. Such low losses are required as a 29cm-long coupler fiber has to guide light from the coupler all the way to the bolometer detector. Finally we show that the coupler fiber dispersion becomes low enough ($\lesssim 1$ps/(THz$\cdot$cm)) only for frequencies below 0.25THz; as dispersion is directly related to the pulse spreading, low dispersion is necessary in order to maintain high signal to noise ratio when transmitting over a significant distance. From these considerations we conclude that the coupler fiber and evanescent coupler considered in this work act as the low-pass filters with a cut-off frequency in the 0.3THz range. Considering that the THz emission spectrum falls off rapidly below 0.25THz, the data presented in this paper can be considered as a measurement of the fiber transmission losses in the near vicinity of 0.3THz.

4. Experimental results and discussion

The power attenuation of porous and non-porous subwavelength THz fibers of different diameter were measured using the directional coupler method. Each test fiber was first aligned with the focal point of the PM2 parabolic mirror for input coupling and then brought parallel to the 7 cm straight segment of a coupler fiber. After every attenuation measurement, the test fiber was removed and the reference signal was acquired by displacing the coupler setup. Fig. 4 presents the measured power attenuation as well as the data fits for each fiber type. Note slight variation in the intensity of the reference signal from one measurement to another. We attribute
such variations to the minor displacements of the coupler fiber, as well as small changes in the knot structure after the two subwavelength fibers were brought in mechanical contact with each other.

Fig. 4. Fiber loss measurements using the directional coupler method. Data is presented for all the porous and rod-in-the-air fibers of different diameters shown in Fig. 1. Plots on the left show the Total and Reference signals measured with and without the test fiber. Plots on the right show fits of the fiber attenuation data ($P_{\text{Fiber}} = P_{\text{Total}} - P_{\text{Ref}}$). a) Non Porous PE fibers. b) Porous PE/Tube fibers produced by straw stacking. c) Porous PE/PMMA fibers produced by dissolving the PMMA rods.

Fig. 5 compiles the results presented in Fig. 4 by showing the measured attenuation coefficient for fibers of different types as a function of the fiber diameter. Error bars correspond to the maximum likelihood limits of the fitted slopes considering the errors bars on the points of Fig. 4 as standard deviations of normal distributions. From Fig. 5 we observe that all but one porous fibers exhibit smaller transmission losses than the rod-in-the-air fibers. We find that losses of porous fibers increase as their diameter decreases.

To rationalize the results presented in Fig. 5 we note that the measured attenuation coefficients are indicators of the total propagation loss which includes both material absorption and scattering on fiber imperfections. In our earlier work\textsuperscript{3} we showed that fiber porosity leads to additional reduction in the subwavelength fiber absorption losses. Therefore, when comparing
fibers of the same outer diameters, and under the assumption of comparable scattering loss, we expect both porous fiber types to demonstrate losses smaller than those of the non-porous fibers. This overall tendency is confirmed in Fig. 5. Moreover, simulations of the next section suggest that below 0.3THz, absorption loss of a PE/PMMA fiber should be about 10 times smaller than absorption loss of the rod-in-the-air fiber of a comparable diameter. Fig. 5, indeed, supports this conclusion for the largest diameter PE/PMMA fiber.

Another interesting feature found in Fig. 5 is the increase in total fiber loss for smaller fiber diameters. Although one expects the material attenuation loss to decrease when reducing the fiber diameter, this is overcompensated by the increase in the fiber scattering loss. Such losses are due to modal scattering at various fiber defects, such as diameter fluctuations, impurities or kinks, and become more pronounced as the fiber diameter gets smaller. Moreover, for the comparable fiber diameters, modal fields of the porous fibers are significantly more delocalized than the modal fields of a rod-in-the-air fiber. Therefore, for smaller fiber diameters we expect a stronger increase in the radiation loss of porous fibers compared to that for the rod-in-the-air fiber. We believe that this is what is observed for the PE/PMMA fiber, in particular (blue triangles in Fig. 5). The effect of fiber diameter fluctuations on losses of subwavelength THz fibers has indeed been documented. By better controlling the fabrication process of porous fibers it should be possible to reduce the amount of fiber defects and, thus, reduce the radiation loss contribution.

From Fig. 5 we, therefore, conclude that all our measurements for the porous fibers are done in the regime where radiation loss becomes important. We also predict that when increasing the diameter of porous fibers the radiation contribution to the total modal loss should decrease, while absorption loss should take over due to the onset of stronger modal confinement in the larger diameter fiber cores. For non porous fibers we do observe in Fig. 5 the transition from a radiation dominated to the absorption dominated regime for the fiber diameters around 300 – 350μm. In fact, to support this statement we have performed one additional loss measurement.
(not shown on Fig. 5) for a non porous fiber of diameter $563 \mu m$ for which the loss of $1 cm^{-1}$ was measured. We would also like to mention that in the literature there are several widely different reported losses for bulk PE, ranging from $\sim 0.2 cm^{-1}$ in Ref. $21$ to $\sim 2 cm^{-1}$ in Ref. $17$. Our measurements on non-porous fibers seem to support the value of the bulk PE loss of $\gtrsim 1 cm^{-1}$.

We now comment on the relative losses of the two porous fiber types. From the fiber cross-sections we estimate $d/\Lambda \sim 0.32 - 0.48$ or porosity $\sim 8 - 18\%$ for the PE/Tube fibers, and $d/\Lambda \sim 0.61 - 0.76$ or porosity $\sim 29 - 45\%$ for the PE/PMMA fibers. The higher porosity leads us to expect lower absorption losses from the PE/PMMA fibers than from the PE/Tube fibers. From Fig. 5 we see that this conclusion holds for the PE/PMMA fiber of $380 \mu m$ in diameter, while for the fibers of smaller diameters, losses of the PE/PMMA fibers are higher than those of the PE/Tube fibers. Although further studies are still needed to resolve the exact nature of this finding, we attribute it to higher radiation losses of the PE/PMMA fibers due to their higher porosity and stronger mode delocalization.

Finally, in passing, we comment on the bending loss of porous fibers. Qualitatively, porous subwavelength fibers are highly sensitive to bending, similarly to their rod-in-the-air counterparts. Theoretical simulations $14$ have shown, however, that bending loss of the porous fibers can be significantly smaller than that of the rod-in-the-air fibers (when comparing fibers of similar absorption losses). Experimentally, it is exceedingly difficult to quantify the bending loss of a subwavelength fiber because of the modal evanescent fields that are strongly delocalized outside of a fiber core. Particularly, study of bending loss as a function of bending radius would require wrapping long (over several meters for subwavelength fibers) and uniform lengths of fiber around circular mandrels of different radii. However, optical contact between a subwavelength fiber and a mandrel would lead to prohibitively high absorption and radiation losses due to interference of the mandrel structure with evanescent fields of the fiber modes. At this time, we are still developing an experimental strategy for measuring bending loss of the subwavelength fibers, and therefore, we did not report fiber bending loss study in this paper.

Moreover, it can be also argued that for many applications of subwavelength fibers, their bending loss could be a less important characteristics than, for example, a maximum deflection angle in a kink introduced into the fiber structure. Indeed, in imaging applications involving scanning of the fiber tip over the test surface, the fiber remains straight, while angle deflection of the fiber tip is used for scanning. Lu et al., in particular, have carried out a systematic study of loss as function of deflection angle for non porous subwavelength fibers. $22$ Qualitatively, we find that 25cm long fiber pieces can still transmit light even when deflected by as much as $20 deg$.

5. Theoretical modelling of subwavelength fibers

In these last two sections we present theoretical modelling of the fundamental properties of the individual subwavelength fibers and an evanescent fiber coupler. We start by calculating effective refractive indices of the eigen modes of the free-standing porous and rod-in-the-air fibers. Both fibers are assumed to have outer diameters of $380 \mu m$. Fiber material refractive index is that of a low density polyethylene $n = 1.516$. Fiber material absorption loss is assumed to be $1 cm^{-1}$. In addition, porous fibers have 7 air holes of diameter $d_h = 95 \mu m$ in their crossection. Hole-to-hole distance is taken to be $\Lambda = 119 \mu m$.

From our simulations we first find that below $\omega = 0.53 THz$ both subwavelength fibers are single moded. Dispersion relation and loss of a porous fiber are labelled as "porous" in Figs. 6(a,b) and are shown as blue solid lines, while those of a rod-in-the-air fiber are labelled as "rod" and are shown as red solid lines. Loss of a porous fiber becomes $\lesssim 0.01 cm^{-1}$ (100 times less that that of a fiber material) when $\omega \lesssim 0.3 THz$, while in the same frequency range loss of a rod-in-the-air fiber becomes $\lesssim 0.1 cm^{-1}$. As experimentally measured loss of a porous
Fig. 6. Fundamental properties of a directional coupler made of two subwavelength fibers (porous and rod-in-the-air) of 380μm diameter separated by 380μm air gap. Both porous and rod-in-the-air fibers are single moded at ω < 0.53THz. a) Refractive indices of the individual fiber modes and corresponding coupler supermodes. Two of the four coupler supermodes have a cut-off frequency of ω = 0.36THz, therefore below this frequency all the individual fibers and a coupler are single moded. b) Modal losses, assuming fiber material bulk absorption loss of 1cm⁻¹. At any frequency losses of a porous fiber are significantly lower than those of a rod-in-the-air fiber. In a) and b), dotted lines show the theoretical fits of the fiber refractive indices and losses derived using asymptotic formula (2). c) Field distributions in the fundamental modes of porous and rod-in-the-air fibers, as well as field distributions in the coupler supermodes at various frequencies. Observe strong coupling regime for ω ≲ 0.35THz, and weak coupling regime at higher frequencies. Only linearly polarized modes of X polarization are shown (dominant Ex component).

fiber is ~ 0.01cm⁻¹, while that of a rod-in-the-air fiber is ~ 0.1cm⁻¹, this logically indicates that our experimental measurements reflect fiber losses averaged over the low frequency range ω ≲ 0.3THz. Alternatively, we can reach the same conclusion by noticing that to arrive to a bolometer detector, the radiation coupled from the test fiber via evanescent coupler into the coupler fiber has to first travel over 29cm-long rod-in-the-air fiber. To have less than 20dB loss over the length of a coupler fiber (with a total 40dB power budget in our experiment) one requires propagation loss of a coupler fiber smaller than 0.16cm⁻¹, which is achievable at frequencies below 0.28THz. Given that the spectral power of a THz source used in the experiment falls off rapidly below 0.25THz, we conclude that presented modal losses correspond to an averaged...
fiber loss in the relatively narrow $0.25THz - 0.30THz$ range.

In Figs. 6(a,b) in dashed lines we also show theoretical fits of the modal dispersion relations and losses in the region of very low frequencies. As our simulations were performed using a finite element mode solver it was challenging to investigate the region of very low frequencies due to strong delocalization of modes of the subwavelength fibers (hence requiring very large computational cells). Complimentary to the FEM solver results, in the regime of subwavelength guiding $rk_0 \ll 1$, where $r$ is a fiber radius, and $k_0 = 2\pi/\lambda$ is a free space wavevector, dispersion relation of the fundamental $HE_{11}$ mode of a rod-in-the-air fiber can be easily derived using Taylor expansions of the governing eigen value equation (see, for example, eq. (9) of Ref. 23). Particularly, by retaining all the terms up to the second order in a small parameter $rk_0$ we can get:

\[ n_{eff} - n_{cl} = \frac{2}{n_{cl}(rk_0)^2} \exp \left( -\frac{n_c^2 + n_{cl}^2}{2n_{cl}(n_c^2 - n_{cl}^2)} (rk_0)^2 \right). \]

In the expression above, $n_{cl}$ and $n_c$ are the cladding and core refractive indices respectively, and the formula is valid for any refractive index contrast as long as $(n_{eff} - n_{cl})/n_{cl} \ll 1$. Note from Figs. 6(a,b) that (2) fits very well the real part of the modal refractive index of the rod-in-the-air fiber (as calculated using FEM solver) for all the frequencies $\omega < 0.35THz$, except for the lowest frequencies for which FEM results are somewhat unreliable. Moreover, fundamental mode loss (imaginary part of the effective refractive index) is also fitted reasonably well by the same formula in the same spectral region. For the rod-in-the-air fiber we used the same $n_c$, $n_{cl}$ as for the FEM method (see the red dotted curves in Figs. 6(a,b)). Interestingly, dispersion relation and loss of the fundamental mode of a subwavelength porous fiber can also be fitted well with the same asymptotic expression (2). To get a good match, however, one needs to take into account porosity of the fiber core region. This can be done by choosing a smaller value for the core refractive index $n_c$, as well as by assuming a smaller loss of the fiber material. We find that the best fit for the porous fiber dispersion relation and loss is achieved by taking $n_c = 1.24$, and the core material absorption loss $0.5 cm^{-1}$ (see the blue dotted curves in Figs. 6(a,b)).

Finally, by using dispersion relations from the FEM simulations and an asymptotic formula (2) we can calculate fiber dispersion $D[ps/(THz\cdot cm)] = d^2\tilde{\beta}(\omega)/d^2\omega$ (where $\tilde{\beta}(\omega)$ is the
modal propagation constant) for the porous and rod-in-the-air fibers. In Fig. 7(a) fiber dispersion is presented as a function of frequency. We note that fiber dispersion reduces exponentially fast with frequency in the regime of subwavelength guidance. Moreover, in this regime dispersion of the rod-in-the-air fiber is much higher that that of a porous fiber. To put the data shown in Fig. 7(a) in prospective we remind the reader that fiber dispersion is directly related to the pulse spreading. Particularly, assuming a temporal width of an input pulse $\tau_0$, after propagating along the length $L$ of a fiber with dispersion $D$, the output pulse width will be $\Delta \tau = LD/\tau_0$. Assuming that the average pulse power stays the same, spectral power density will decrease roughly inversely proportional to the pulse width due to pulse spreading. This, in turn, can strongly affect a signal to noise ratio. As an example, consider $D = 1 \text{ps/(THz cm)}$ and a 30cm long fiber. For a typical 1ps-long THz input pulse, the output pulse width will then be 30ps, thus decreasing a signal to noise ratio by $\sim 14\text{dB}$. From Fig. 7(a) we see that dispersion of a porous fiber becomes less than $1 \text{ps/(THz cm)}$ when $\omega \lesssim 0.23\text{THz}$, while for the rod-in-the-air fiber it happens when $\omega \lesssim 0.17\text{THz}$. This conclusion further supports our claim that a 29cm-long rod-in-the-air coupler fiber functions as a low pass filter. Indeed, for frequencies higher than 0.2-0.3THz not only the fiber exhibits high attenuation loss, but it also exhibits high dispersion, which leads to further reduction of the signal to noise ratio due to pulse spreading.

6. Theoretical modelling of a directional coupler made of two subwavelength fibers

Finally, we investigate performance of an evanescent coupler operating at low frequencies at which both fibers making the coupler are guiding in a subwavelength regime. In Figs. 6(a,b) in thin solid curves with circles and crosses we present dispersion relations of the coupler supermodes. In what follows we only consider frequencies below 0.53THz at which both porous and rod-in-the-air fibers are single moded. Due to mirror symmetry with respect to the X axis, coupler supermodes can be labelled as X or Y polarized depending whether the dominant component of their electric field is $E_x$ or $E_y$. To indicate the supermode polarization, next to the modal name we also add in parenthesis the leading component of the electric field. Although X and Y polarized modes of a coupler are not degenerate, from Figs. 6(a,b) we see that their dispersion relations and losses remain very similar. In the frequency range $0.36\text{THz} < \omega < 0.53\text{THz}$ there are two types of supermodes. One mode labelled as "$s_{\text{por}}$" is a mode with most of its field localized in the vicinity of a porous fiber. The third row of Fig. 6(c) we present examples of the field distributions in the X polarized fundamental mode of a porous fiber, as well as X polarized $s_{\text{por}}$ supermodes of a coupler at three different frequencies. Interestingly, this supermode has a cutoff at $0.36\text{THz}$ for both polarizations (see blue curves with crosses and circles in Fig. 6(a)). Therefore, below $0.36\text{THz}$ the coupler becomes single moded.

The second supermode of a coupler labelled as "$s_{\text{rod}}$" is a mode with most of its field localized in the vicinity of a rod-in-the-air fiber. In the second row of Fig. 6(c) we present examples of the field distributions in the X polarized $s_{\text{rod}}$ supermodes at three different frequencies. Notably, this supermode is a fundamental mode of a coupler and does not have a cutoff. Although, in principle, one of the polarizations of the $s_{\text{rod}}$ supermode can have a cutoff at lower frequencies, in the $0.20\text{THz} < \omega < 0.36\text{THz}$ frequency range the two polarizations are almost degenerate (see red thin curves with crosses and circles in Fig. 6(a)). From the second row of pictures in Fig. 6(c) we also conclude that when going to lower frequencies, the field of the $s_{\text{rod}}$ supermode gets strongly expelled from the core of a rod-in-the-air fiber. As a result, the field starts having a substantial overlap with the porous fiber, thus leading to strong hybridization of the $s_{\text{rod}}$ supermode.

The last question that we want to address is about the coupling efficiency of an evanescent coupler studied experimentally in this paper. We first introduce the following notations. $|\text{Por}^+\rangle$ and $|\text{Por}^-\rangle$ signify the transverse electromagnetic fields of the forward and backward propa-
gating modes of a porous fiber, \(|s_{\text{rod}}^+\rangle\) and \(|s_{\text{rod}}^-\rangle\) are the transverse electromagnetic fields of the forward and backward propagating supermodes of a coupler, and \(|\text{Rod}^+\rangle\) are the transverse electromagnetic fields of the forward propagating mode of a rod-in-the-air fiber. The transverse and longitudinal fields of the forward and backward propagating modes are related as follows:

\[
(E_x^+, E_z^+, H_y^+, H_z^+) = (-E_x^-, E_z^-, H_y^-, H_z^-).
\]

We use the \(\exp(\beta z - i \omega t)\) convention for the \((t,z)\) dependence of the modal fields. Finally, \(R\) denotes reflection coefficient at the coupler input end, \(T\) denotes transmission coefficient through the coupler, and \(L\) denotes coupler length. We now write two equations for the continuity of the transverse electromagnetic fields at the interface between the input porous fiber and a coupler, as well as at the interface between the coupler and the output rod-in-the-air fiber:

\[
\begin{align*}
\text{Input end:} & \quad |\text{Por}^+\rangle + R |\text{Por}^-\rangle = A |s_{\text{rod}}^+\rangle + B |s_{\text{rod}}^-\rangle \\
\text{Output end:} & \quad A \exp(i\beta_{\text{rod}} L) |s_{\text{rod}}^+\rangle + B \exp(-i\beta_{\text{rod}} L) |s_{\text{rod}}^-\rangle = T |\text{Rod}^+\rangle
\end{align*}
\]  

(3)

We now use the orthogonality relations for the modes of lossy waveguides \(\langle \psi_\beta | \psi_\beta' \rangle \sim \delta(\beta - \beta')\), where the dot product between the two modes is understood as (Ref. 24 p. 115):

\[
\langle \psi_\beta | \psi_\beta' \rangle = 1/4 \int dxdy [E(x,y)_{\beta,\beta'} \times H(x,y)_{t,\beta} - E(x,y)_{t,\beta} \times H(x,y)_{\beta,\beta'}].
\]  

(4)

From (4) we can then write that \(|s_{\text{rod}}^+\rangle |s_{\text{rod}}^-\rangle = 0, \langle \text{Por}^+ | \text{Por}^- \rangle = 0\). To solve for the supermode coefficients \(A\) and \(B\) we multiply the left and right sides of the first equation in (3) by \(|s_{\text{rod}}^\pm\rangle\) and use the above mentioned orthogonality relations. Thus obtained expressions for \(A\) and \(B\) will still contain the reflection coefficient \(R\). By substituting \(A\) and \(B\) back into the first equation in (3), and multiplying its left and right sides by \(\langle \text{Por}^- \rangle\) one can show that in this formulation \(R = 0\). Finally, by substituting thus found \(A\) and \(B\) coefficients into the second equation of (3), and by multiplying its left and right sides by \(\langle \text{Rod}^+ \rangle\) we get the following expression for the transmission coefficient:

\[
T = \exp(i\beta_{\text{rod}} L) \eta^+ + \exp(-i\beta_{\text{rod}} L) \eta^-
\]

\[
\eta^+ = \frac{\langle s_{\text{rod}}^+ | \text{Por}^- \rangle |\text{Rod}^+\rangle}{\langle s_{\text{rod}}^+ | s_{\text{rod}}^- \rangle |\text{Rod}^+\rangle |\text{Rod}^-\rangle}, \quad \eta^- = \frac{\langle s_{\text{rod}}^- | \text{Por}^- \rangle |\text{Rod}^+\rangle |\text{Rod}^-\rangle}{\langle s_{\text{rod}}^- | s_{\text{rod}}^+ \rangle |\text{Rod}^+\rangle |\text{Rod}^-\rangle}.
\]  

(5)

Assuming that all the modes are normalized to carry a unit power (meaning \(1/2 \int dxdy \cdot \mathbf{E}(x,y) \times \mathbf{H}^*(x,y) = 1\)), the total power transmission through the coupler is given by \(|T|^2\).

From our calculations we observe that \(|\eta^-| \ll |\eta^+|\), therefore, transmission coefficient can be written in a simpler form:

\[
|T|^2 \simeq \exp(-2 \text{Im}(\beta_{\text{rod}} L)) \cdot |\eta^+|^2.
\]  

(6)

From this expression it follows that transmission through the subwavelength evanescent coupler is proportional to the coupler efficiency \(|\eta^+|^2\) determined by (5). In Fig. 7(b) we present the coupler efficiency as a function of wavelength. Due to strong modal delocalization at lower frequencies we observe a fast increase in the coupler efficiency to above 50% at \(\omega \lesssim 0.28 THz\). This is yet another argument in favor of our earlier conclusion that the coupler considered in this work operates as a low pass filter with a cutoff frequency \(\sim 0.3 THz\).

7. Conclusions

We have designed, fabricated and characterized the THz guiding properties of subwavelength porous polyethylene fibers consisting of a polymer rod containing an array of air holes. Compared to losses of the rod-in-the-air subwavelength fibers, losses of the porous fibers of comparable diameters were shown to be as much as 10 time smaller. The fiber attenuation coefficient
was measured by a novel and non-destructive directional coupler method. In the vicinity of 0.3THz, the loss of a 380μm diameter porous fiber with ~ 40% porosity was measured to be 0.01cm⁻¹ = 4.34dB/m. Finally, we present theoretical studies of the optical properties of individual subwavelength fibers and a directional coupler. From these studies we conclude that coupler setup studied in this paper also acts as a low pass filter with a cutoff frequency around 0.3THz. Considering that the spectrum of a terahertz source used in this work falls off rapidly below 0.25THz, the reported loss measurements are, thus, the bolometer averages over the ~ 0.25THz – 0.3THz region.

Acknowledgments

This work is supported in part by the Canada Research Chair program and the Canada Institute for Photonic Innovations project FP3.