CONVERSION OF THE MODIFIED KOVÁCS MODEL PARAMETERS TO THE BROOKS & COREY AND VAN GENUCHTEN MODEL PARAMETERS FOR GRANULAR SOILS

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ABSTRACT
The modified Kovacs (MK) model can be considered as one of the most versatile predictive models available to estimate the unsaturated hydraulic functions. The MK model relies on basic geotechnical properties to obtain the water retention curve (WRC), which can then be used to define the relative permeability function. In addition, the model can also take into consideration hysteresis effects and volume changes. However, the MK model has not yet been introduced in many codes, as most numerical applications needing a mathematical equation to represent the WRC of soils use the Brooks & Corey (BC) (1964) and/or the van Genuchten (vG) (1980) models. To allow for a wider application of the MK model, the authors propose in this paper a direct conversion of MK model parameters to the BC and vG parameters for granular soils taken from the Grizzly soil data base (Haverkamp et al. 1998). The proposed equations were developed using data from 15 granular soils. Results show that BC and vG parameters can be deduced adequately from MK model parameters.

1 INTRODUCTION
Description and prediction of water flow through unsaturated soils imply an understanding of unsaturated soil properties. One of the most important properties in this regard is the water retention curve (WRC), which relates the volumetric water content $\theta$ (cm$^3$/cm$^3$) to the suction $\psi$ (cm of water, or kPa). WRCs are often used to derive other unsaturated functions that are difficult to measure, such as the permeability functions (relationship between hydraulic conductivity and suction), thermal conductivity functions (relationship between thermal conductivity and suction), and shear strength functions (e.g. Fredlund 2006). When experimental measurements of $\theta$ vs $\psi$ are available for a given soil, the data can be fitted using descriptive equations to obtain a continuous function over the entire range of suction. To this end, the descriptive models proposed by Brooks and Corey (1964) and van Genuchten (1980) are often used (called herein after BC and vG models, respectively).

The measurement of the WRC in the laboratory and in the field can be relatively time consuming and expensive. In some particular situations (for example at the pre-feasibility stage of a project), it may thus be useful to have estimates of the WRC before conducting the required suction tests for the conditions encountered and expected in the field. In this regard, various predictive models (sometimes called pedo-transfer functions – PTF) have been developed to estimate the WRC using routinely available data such as particle size distribution, porosity, organic matter content, and clay content.
Two main types of model have been developed to predict WRC from basic geotechnical properties (Haverkamp et al. 1999): i) functional regression models, and ii) physically based models.

Functional regression models propose relationships (using multiple-linear regression or artificial neural networks) between parameters of descriptive (best-fitting) models such as the BC and vG models and various soil properties (grain-size, porosity, and other textural-structural soil data) to predict the entire WRC. Many models of this type have been proposed over the years (e.g. Clapp and Hornberger 1978; McCuen 1981; Cosby et al. 1984; Rawls and Brakensiek 1985; Wösten and van Genuchten 1988; Vereecken et al. 1989). However, the applicability validity of the ensuing parameters is in general restricted to the soil tested in each study (Haverkamp et al. 1999) since this type of model typically ignores the physics that govern the water retention in a soil. But in spite of this limitation, functional regression models offer a good correlation between the shape parameter of the WRC and cumulative particle-size distribution function (Jauhiainen 2004; Haverkamp et al. 1999). This shape similarity was used as the main hypothesis for the physically based model proposed by Arya and Paris (1981). This type ii model first translates a particle-size distribution into a pore-size distribution. Then, the cumulative pore volumes corresponding to progressively increasing pore radii are divided by the sample bulk volume to give the volumetric water content, and the pore radii are converted to equivalent soil water pressures using the equation of capillarity. A somewhat similar model was later proposed by Haverkamp and Parlange (1986) and tested on a coarse-textured sand without organic matter. The proposed model allows direct estimation of the Arya and Paris (1981) model parameters. This method has the advantage of interpreting the cumulative particle size distribution function in its continuous form (Haverkamp et al. 1999).

Later, Haverkamp et al. (2005) proposed an improved physically-based approach to estimate the WRC parameters from textural soil properties. The proposed method relies upon the concept of shape similarity and uses the method of geometrical scaling.

Another model that can be added to the physically-based type was proposed by Kovács (1981). This model makes a distinction between capillary and adhesive forces, both acting simultaneously to induce suction. This approach potentially provides a conceptual view that can be related to the actual processes involved (e.g., Celia et al. 1995; Nitao and Bear 1996). However, the Kovács (1981) model, in its original form, is not easy to use because some of the key parameters were not completely defined. Modifications to the model proposed by Aubertin et al. (1998, 2003) have lead to practical predictions of the WRC for granular materials knowing only the void ratio \( e \) and grain-size parameters; it can also be used for some clayey soils. The ensuing modified Kovács (MK) model has evolved over the years and it can now take into consideration hysteresis effects (Maqsoud et al. 2006) and volume changes (Mbonimpa et al. 2006b) in the prediction of the WRC.

In the common practice, for analytical derivations or for numerical studies, some WRC expressions are more often used than the others. Such is the case with the Brooks and Corey (1964) formulae as it is simple and convenient to use for analytical derivations. On the other hand, many commercial codes that simulate water flow through unsaturated soils use the van Genuchten (1980) expressions, as it represents well the WRC of a wide variety of soils. Hence, it can be convenient during the preliminary phase of a project, when suction tests results are not yet available, to convert MK model parameters (a predictive model) to Brooks and Corey (BC) and/or to van Genuchten (vG) parameters (two descriptive models). This paper presents such simple and practical conversion relationships for granular soils. Soil properties from the literature are first introduced into the MK model to obtain predicted WRCs. These curves are then fitted with the BC and vG models to obtain the optimal parameters. Finally, simple relationships between the MK model parameters and the BC and vG models parameters are proposed. It is important to recall at the onset that the MK model has previously been shown to correctly predict the WRC of most granular soils (see Maqsoud et al. 2006 and Mbonimpa et al. 2006b for this validation). There is no additional assessment of the predictive capabilities of the MK model here; the paper focussed only on the relationships between the values of the different WRC models parameters.

2. FORMULATIONS

In the following, three different models are presented: the descriptive BC and vG models and the predictive (physically based) MK model.

2.1 Brooks and Corey model

The BC model can be expressed as:

\[
\theta = \theta_s \quad \text{if } 0 \leq \psi \leq \psi_{aBC}
\]

\[
\theta = \theta_r + (\theta_s - \theta_r) \left[ \frac{\psi}{\psi_{aBC}} \right]^{-\lambda} \quad \text{if } \psi > \psi_{aBC} \tag{1}
\]

where \( \lambda \) is the pore-size distribution index, \( \psi_{aBC} \) is the air entry value of the BC model, \( \theta_r \) is the residual volumetric water content, and \( \theta_s \) is the saturated volumetric water content. \( \theta_r, \theta_s \) and \( \psi_{aBC} \) were estimated with the RETC code which uses a combination of linear regression and a nonlinear optimization method (van Genuchten et al. 1991).

2.2 Van Genuchten model

Various sigmoidal curves have been used to describe soil water retention data, including the popular model of van Genuchten (1980). The vG model proposes the following function to represent the WRC between the \( \theta_s \) and \( \theta_r \):

\[
\theta = \theta_r + (\theta_s - \theta_r) \left[ 1 + (\alpha_v \psi)^{n_v} \right]^{-m_v} \tag{2}
\]
where \(a_{\text{vG}}, n_{\text{vG}}\) and \(m_{\text{vG}}\) are empirical parameters. The value of \(1/a_{\text{vG}}\) can be considered as an approximation of the air entry value (\(\psi_d\)), particularly when the ratio \(m/n\) is small (Cresswel et al. 2006). Equation [2] can be used with \(m_{\text{vG}}\) and \(n_{\text{vG}}\) as independent variables although a unique relation between \(m_{\text{vG}}\) and \(n_{\text{vG}}\) is more commonly assumed. Here, \(m_{\text{vG}} = 1 - \frac{2}{n_{\text{vG}}}\) is used, as proposed by van Genuchten (1980).

2.3 Modified Kovács model

The modified Kovács (MK) model is a predictive model that can be described by a set of equations. The basic equation of the model can be written as follows (Aubertin et al. 1998):

\[
\theta = n[1-(1-S_c)(1-S_e)]
\]  

where \(\theta\) is the volumetric water content, \(n\) is the total porosity, \(S_c\) and \(S_e\) correspond to adhesive and capillary components of \(\theta\), and \(\langle \rangle\) represents the Macauley brackets \((\langle y \rangle = 0.5(y+1))\).

The contributions of \(S_c\) and \(S_e\) to the total volumetric water content are expressed as functions of the equivalent capillary rise \(h_{co}\) (cm) and suction \(\psi\):

\[
S_c = 1 - [h_{co} / \psi^2 + 1]^m \exp[-m(h_{co} / \psi^2)]
\]

\[
S_e = a_c \left(1 - \ln(1 + \psi / \psi_o) / \ln(1 + \psi_o / \psi_o) \right) \left(1 + \psi / \psi_o \right)^{2/3} e^{1/3(\psi / \psi_o)^1/6}
\]

where

\[
h_{co} = \frac{0.75 \cdot \cos \beta_d}{e^{D_10^{-6}} \cdot 1.17 \cdot \log(C_U) + 1}
\]

In equation [6], \(D_{10}\) is the diameter (cm) corresponding to 10% passing on the cumulative grain-size distribution, \(\beta_d\) corresponds to the contact angle taken as zero (for drying conditions), \(C_U\) is the coefficient of uniformity \((C_U = D_{60} / D_{10})\), and \(e\) is the void ratio. In equation [4], \(m\) (-) is a pore size coefficient that controls capillary saturation; its value is expressed as a function of the grain size distribution. In equation [5], \(a_c\) (-) is the adhesion coefficient, \(\psi_o\) is the residual suction (cm), and \(\psi_o\) is a normalizing parameter for unit consistency \((\psi_o = 1\ \text{cm when } \psi < \psi_{co}\) are given in cm).

For granular (non cohesive, low plasticity) materials, the value of the pore-size distribution parameter \(m\) can often be closely approximated by the inverse of the uniformity coefficient \((m = 1/C_U)\), while the adhesion coefficient \(a_c\) is approximately constant \((a_c = 0.01, \text{ when suctions are in cm})\). For more details on the MK model, the interested reader can refer to Aubertin et al. (1998, 2003).

3 PARAMETERS CONVERSION

3.1 Data used for comparative evaluation

Data obtained on different granular soils were used to develop the parameters conversion between the MK model and the vG and BC models. Overall, 15 granular soils were chosen from the Grizzly soil database (Haverkamp et al. 1998), which cover a wide range of soil properties. The WRCs of some of these soils were previously predicted correctly from the MK model (Mbonimpa et al. 2006a; Maqsoud et al. 2006). For each soil, parameters \(D_{10}\), \(D_{60}\) and \(n\) were available, so \(h_{co}\) was calculated for each one using equation [6]. Table 1 gives information on these soils. It shows that the range of porosity is between 0.32 and 0.43, \(D_{10}\) is between 4.9x10^{-5} and 16.8x10^{-5} cm, \(D_{60}\) between 16.7x10^{-3} and 36.2x10^{-3} cm and \(h_{co}\) is falls between 51 and 162 cm.

The data were processed using the approach proposed in Figure 1. First, the WRC was estimated for each soil using the MK model. The predicted curves were then fitted with the vG and BC models (using the RETC code); the best fitted parameters for each model were determined along with the coefficient of determination \(R^2\) (van Genuchten et al. 1991).

![Figure 1: Steps for predicted parameters determination](Image)

For all curves, the residual water content \(\theta_r\) was considered nil (an acceptable simplification for coarse soils) and the saturated volumetric water content \(\theta_s\) was taken equal to the total porosity \(n\) (common assumption for drainage curves).

Results of the fitting process using the RETC code are also presented in Table 1. One can see that \(R^2\) is high, usually between 0.97 and 0.99 for both descriptive models. The air entry values for the different soils are slightly different depending on the model. Values are generally higher when using the vG model (assuming that 1/\(a\) is close to \(\psi_d\)). This is due to the different shapes of the curves. Nonetheless, all the values fall between 17...
and 47 cm of water (typical AEV for relatively coarse granular soils).

3.2 Equations for parameters conversion

The first step to establish the parameters conversion from one model to the other is to evaluate the correlation between the different basic parameters for each model. Similar correlation analyses were used by Schaap et al. (1998) and Haverkamp et al. (2005) to establish links between soil parameters and hydrogeological properties.

A matrix correlation between the different parameters (see Table 2) was calculated using STATITCF software (ITCF 1991). The formula used here for the correlation calculation is:

\[ r = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum (x_i - \bar{x})^2 \sum (y_i - \bar{y})^2}} \]  

In equation [7], \( \bar{x} \) is the mean value of the \( x_i \) values and \( \bar{y} \) is the mean value of the \( y_i \) values.

Results of the correlation analyses are presented in Table 2. A value of \( r \) greater than 0.56 (Mangin 1975) usually indicates a strong correlation between two parameters.

### Table 1: Soils identification, basic geotechnical parameters and fitted parameters

<table>
<thead>
<tr>
<th>Soil</th>
<th>Modified Kovács Model</th>
<th>van Genuchten Model</th>
<th>Books and Corey Model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>n</td>
<td>( D_{10} ) (x10(^{-3}) cm)</td>
<td>( D_{60} ) (x10(^{-3}) cm)</td>
</tr>
<tr>
<td>Soil 6</td>
<td>0.384</td>
<td>16.80</td>
<td>36.19</td>
</tr>
<tr>
<td>Soil 7</td>
<td>0.354</td>
<td>14.55</td>
<td>35.00</td>
</tr>
<tr>
<td>Soil 8</td>
<td>0.430</td>
<td>15.49</td>
<td>26.30</td>
</tr>
<tr>
<td>Soil 9</td>
<td>0.400</td>
<td>15.30</td>
<td>24.01</td>
</tr>
<tr>
<td>Soil 10</td>
<td>0.385</td>
<td>15.30</td>
<td>24.00</td>
</tr>
<tr>
<td>Soil 12</td>
<td>0.370</td>
<td>14.77</td>
<td>34.77</td>
</tr>
<tr>
<td>Soil 17</td>
<td>0.433</td>
<td>4.31</td>
<td>19.64</td>
</tr>
<tr>
<td>Soil 25</td>
<td>0.391</td>
<td>16.50</td>
<td>23.12</td>
</tr>
<tr>
<td>Soil 30</td>
<td>0.322</td>
<td>6.77</td>
<td>16.72</td>
</tr>
<tr>
<td>Soil 47</td>
<td>0.411</td>
<td>4.90</td>
<td>18.66</td>
</tr>
<tr>
<td>Soil 70</td>
<td>0.317</td>
<td>4.70</td>
<td>26.40</td>
</tr>
<tr>
<td>Soil 71</td>
<td>0.371</td>
<td>6.15</td>
<td>29.42</td>
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<tr>
<td>Soil 72</td>
<td>0.416</td>
<td>4.89</td>
<td>25.73</td>
</tr>
<tr>
<td>Soil 73</td>
<td>0.394</td>
<td>5.98</td>
<td>25.14</td>
</tr>
<tr>
<td>Soil 74</td>
<td>0.366</td>
<td>5.47</td>
<td>26.80</td>
</tr>
</tbody>
</table>

### Table 2: Coefficients of correlation r between basic geotechnical parameters and fitted parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Modified Kovács Model</th>
<th>van Genuchten Model</th>
<th>Books and Corey Model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( h_{co} )</td>
<td>( D_{10} ) (x10(^{-3}) cm)</td>
<td>( D_{60} ) (x10(^{-3}) cm)</td>
</tr>
<tr>
<td>( h_{co} )</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>( D_{10} )</td>
<td>-0.91</td>
<td>-0.55</td>
<td>-0.87</td>
</tr>
<tr>
<td>( D_{60} )</td>
<td>-0.51</td>
<td>+0.47</td>
<td>+0.73</td>
</tr>
<tr>
<td>( 1/\alpha_{vg} )</td>
<td>+0.05</td>
<td>+0.28</td>
<td>+0.89</td>
</tr>
<tr>
<td>( n_{vg} )</td>
<td>-0.85</td>
<td>+0.28</td>
<td>+0.89</td>
</tr>
<tr>
<td>( \psi_{ABC} )</td>
<td>+0.89</td>
<td>-0.75</td>
<td>-0.78</td>
</tr>
<tr>
<td>( \lambda_{BC} )</td>
<td>-0.94</td>
<td>+0.93</td>
<td>+0.93</td>
</tr>
</tbody>
</table>

For the vG model, data in Table 2 shows that \( 1/\alpha_{vg} \) is correlated positively to \( h_{co} \) and negatively to \( D_{60} \). Also, \( n_{vg} \) is correlated negatively to the \( h_{co} \) and positively to the \( D_{10} \). For the BC model parameters, one can see in Table 2 that \( \psi_{ABC} \) is correlated positively to \( h_{co} \) and negatively to \( D_{60} \). Also, \( \lambda_{BC} \) is correlated positively to the \( D_{10} \) and negatively to \( h_{co} \). Table 2 suggest that \( 1/\alpha_{vg} \) and \( \psi_{ABC} \) may be predicted using \( h_{co} \) and \( D_{60} \), while \( n_{vg} \) and \( \lambda_{BC} \) can be predicted using \( h_{co} \) and \( D_{10} \).

The Lab Fit curve Fitting Software (Wilton and Cleide Pereira 2007) was used to establish mathematical equations that best relate the various parameters. These equations can be expressed as follow:

\[ 1/\alpha = 19.57 \times \left( h_{co} \right)^{2.8E-03} / D_{60} \]  

\[ n_{vg} = 1.94 \times D_{10} \left( -9.19 / h_{co} \right) \]  

\[ \psi_{ABC} = 1.51 \times \left( h_{co} \right)^{3.45E-01} / D_{60} \]  

\[ \lambda_{BC} = 4.9E-01 \times D_{10} \left( -10.84 / h_{co} \right) \]

4 COMPARISON BETWEEN CALCULATED AND FITTED DATA

The calculated (with eqs 8-11) and originally fitted parameters are compared in Figures 2 to 5. These figures show a high correlation between calculated and fitted.
parameters, with $R^2$ between 0.83 and 0.93 and a slope of nearly 1; a perfect correlation would lead to a regression line with a slope of 1 with all points on the line. Hence, one can consider that vG and BC parameters can be estimated adequately using the MK model parameter $h_{co}$, with the $D_{10}$ and $D_{60}$.

The parameters of the BC and vG models obtained from the proposed conversion equations can be integrated in the main equations (eq. [1] and [2]) to calculate the WRC. This is shown for the different soils used in this study (BC Cal. and VG Cal.) in Figure 6 and 7. These calculated WRCs are also compared to the fitted WRC (see BC Fitted and vG Fitted) in Figure 6 and 7.

In Figure 6, one can observe that there is a good agreement between calculated (using conversion equations) and fitted water retention curves. This indicated that the predictive equations presented here give realistic parameter values for the studied granular soils.

For soils included in Figure 7, the predicted WRCs using the MK model show a slight difference when compared with the RETC fitted and with the calculated WRCs from the BC and vG models. These differences can be due to the simplifying for the $\theta_r$ values and the slope of the curves. Nevertheless, considering the limited precision related to the measurement and prediction of the WRC, it can be considered that the proposed conversion of the MK model to BC and vG models parameters gives realistic results for the granular soils analysed here.

5 CONCLUSION

Conversion equations were defined to provide a simple way to convert MK model parameters into BC and vG model parameters for granular soils. A comparison between calculated and fitted WRC showed that proposed equations lead to fairly good estimates of the vG and BC fitting parameters using grain-size distribution parameters $D_{10}$ and $D_{60}$ and the equivalent capillary rises ($h_{co}$) used in the MK model. The proposed conversion parameters are particularly convenient at the first stage of a project when little information is available. More work is presently underway to extend the conversion to other soil types and other descriptive models.

ACKNOWLEDGEMENTS

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Figure 6: Comparison between predicted WRC using the MK model, fitted and calculated WRCs using the BC and vG model for soil 6, 7, 8, 9, 10, 12 and 25 (from Grizzly database)
Figure 7: Comparison between predicted WRC using the MK model, fitted and calculated WRCs using the BC and vG model for soil 17, 30, 47, 70, 71, 72, 73 and 74 (from Grizzly database)
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